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SOVIET ASTRONOMY

AJ

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FORTIETH ANNIVERSARY OF SOVIET ASTRONOMY

The fortieth anniversary of the Great October Revolution is approaching. All our country is taking stock of the results of reconstruction of our public and government life.

This reconstruction amounts to a transformation of Russia from a backward agricultural state, whose resources were systematically squandered by foreign firms, into a great economically independent industrial country, and this has had a very great impact upon the development of science in general and astronomy in particular.

Astronomy is an observational science whose development depends upon the availability of diverse material concerned, in the great majority of cases, with very weak objects in our galaxy and the even more extended metagalactic system, and this in its turn depends upon the state of our optico-mechanical industry. As is well known, before the Revolution the optico-mechanical industry was virtually nonexistent in our country. Each astronomical instrument, each assembly, even down to small astronomical pavilions, had to be ordered from abroad.

The creation of an optico-mechanical industry was begun with the setting up of the State Optical Institute and a number of other undertakings which gave a powerful stimulus to the development of constructional activities in scientific establishments in the Soviet Union. This creative and constructive work has taken us a long way since the first 13" reflector built by the late Astronomical Institute in Leningrad for the newly organized Abastuman Observatory in the Georgian SSR if one remembers the new and original optical and photoelectric systems, meniscus telescopes, large aperture nebular spectrographs, isophotometers, automatic electrophotometers and many other instruments which are now available. Soviet astronomy is justly famous abroad. One may mention, for example, the tower solar telescope, the so-called Einstein tower at Potsdam, which is equipped with Soviet diffraction gratings and whose exceptional quality has been pointed out by Potsdam astronomers. A large number of the stations designed for optical observations of the Earth's artificial satellites are, in a number of countries, equipped with instruments produced in the Soviet Union.

At the present time Soviet optico-mechanical industry has begun the construction of huge telescopes which will further increase the range of Soviet astronomy.

From the first years of the Revolution, steps were taken to develop observational astronomy, particularly astrophysics, and this has led to the establishment of a large number of new observatories. It is common knowledge that before the Revolution professional astronomy was mainly represented at Pulkovo and its two branches at Simeiz and Nikolaevo. Almost all the other observatories were closely associated with universities, and were situated in towns; their staffs, being small, had to spend most of their time teaching in the universities.

Naturally, one of the main goals of Soviet astronomy was to establish new observatories situated in more suitable places in the Southern districts of our country. This process of organization of observational astronomy was unusually difficult and was not everywhere equally successful. In the process of its development the Seimez Observatory soon became an independent institution and subsequently, after the Great Patriotic War, it developed into a powerful center of astrophysics situated around the Partizanskoe village in central Crimea. The Pulkovo Observatory,which was destroyed during the war,has been rebuilt on a larger scale. In many Union republics, namely, Armenia, Kazakhstan, Tadzhik SSR, Turkmenistan, Uzbek SSR, Ukraine, and recently in Azerbaidzhan, astronomical observatories have been set up and were organized with the help of our central institutions. The Moscow Astronomical Observatory was moved to a new place in Lenin Hills and was equipped with new buildings and instruments. Special observatories have been organized, for example, in Poltava,where determinations of latitude and gravimetric work are being carried out, in the mountainous districts of Kislovodsk and Alma-Ata where the sun is being studied, etc.

In parallel with these developments, the staffs of the observatories continued to increase. The main astronomical observatory at Pulkovo had a staff of four people when it was established in 1839,and immediately before the Revolution its staff consisted of 28 people. During the years of Soviet rule its scientific personnel increased to some hundreds of people. The vigor of the astronomical community in the USSR and the diversity of the work carried out by its members is in a considerable contrast with other countries in which astronomy is well developed. For example, in France, where astronomical work is at a very high level, a typical staff of an astronomical observatory consists of 3-4 people and only the Paris and Meudon Observatories and the Astrophysical Institute in Paris have staffs of about 30 scientists.

The continuous growth of astronomical work in the USSR requires an increase in the volume of periodical publications printed in the Union. The Russian language has become essential to every foreign astronomer and is being studied by many specialists abroad.

It is essential to remember that this enormous growth of Soviet astronomy was greatly helped by the spirit of collectivism which is characteristic of Soviet society. Thus, for example, already during the first years of Soviet rule both Leningrad and Moscow astronomers have together searched for places suitable for astronomical observatories. Separate projects, as well as programs of observations during total solar eclipses,were also carried out collectively with a preliminary discussion of the method of observation and with a simultaneous ordering of new apparatus for all the institutions taking part.

Since 1937 this cooperation has taken the form of a completely organized collaboration independent of departmental limits and this is now directed by the Astronomical Council of the Academy of Sciences of the USSR and its numerous committees concerned with separate branches of the subject. This has been very helpful in the organization of long-range projects such as, for example, the setting up of a catalogue of weak stars in a time-independent coordinate system which was produced as a result of a collaboration between a large number of Soviet observatories and some foreign ones.

We cannot discuss here all the separate investigations which have been carried out, whatever their importance. We can only sketch out some of the new tendencies in astronomy which have appeared and are developing in our country.

It is well known that in the days before the Revolution our country used exclusively foreign ephemerides which are necessary not only to astronomy but also to many other branches of experimental science, such as geodesy, navigation, etc. The first Soviet astronomical yearbook was worked out in Leningrad, due to the exceptional ability of M. A. Vil'ev, and later astronomical ephemerides began to be systematically compiled at the Astronomical Institute,while at the present time this work is being done at the Institute of Theoretical Astronomy which has the necessary numerous staff and all the technical means, including electronic machines.

During the last 40 years, contemporary astronomy has changed beyond all recognition. In the 19th century and the beginning of the 20th century it was mainly concerned with the study of the motion of the planets and the determination of stellar positions, while later the range of astronomical work was considerably increased to include the whole of the universe and now covers problems which have very great cognitive and philosophical importance.

Soviet astronomy has not been left behind in this overall tendency in science and has frequently produced its own original and valuable contributions. In the USSR, theoretical astrophysics and stellar astronomy, which was nonexistent before, have developed very considerably. Our specialists have obtained important results in many areas of contemporary astronomy, for example, in the interpretation of spectral characteristics of stars and the sun, the luminescence of nebulae and the interstellar galactic medium, the structure and dynamics of the galaxy, the internal composition of stars and planets, the motion of asteroids and satellites of planets, the evolution of stars, the origin of the solar system, and many other problems. Soviet astronomers have taken on international duties in connection with the setting up of catalogues of variable stars which are being continuously supplemented and which are of importance not only in studies of the physical nature of stars in general but also in providing information about the dynamics and the structure of our galactic system.

In recent years we have witnessed in the Soviet Union the development of radio astronomy which is a new method of penetrating into the depths of the universe and which has already produced some important results.

On October 4, 1957 the whole world witnessed a new achievement of Soviet science when, as a result of the persistent effort of Soviet scientists, engineers, technicians, and other workers, the first artificial satellite of the Earth was successfully launched. This important event indicates the great achievements of Soviet scientific thought and opens a new era in the history of world science.

Soviet astronomers take an active part in numerous international meetings and conferences. The meeting of the International Astronomical Union which is expected to take place in Moscow in 1958 will demonstrate still more the successes of Soviet astronomy and will undoubtedly strengthen the friendly collaboration between the astronomers of the whole world who are striving for peaceful cooperation.

SOME RESULTS OF INVESTIGATIONS OF NONSTATIONARY PROCESSES ON THE SUN

A. B. Severnyi

In this paper some results of spectroscopic investigations of nonstationary processes (flares, moustaches, continuous emission), carried out at high resolving power and dispersion during the past four years, are summed up and discussed. Continuous nonstationary emission originates in optically-thin grains (size $0.5\text{-}3''$, lifetime up to 30^m) at the different levels of solar atmosphere ($0 \le \tau \le 0.6$). This fact, as well as the intensity-distribution in the spectrum and the polarization of this emission, indicate that the emission is of nonthermal origin. This emission cannot be caused by recombination processes or by scattering on electronic condensations. Possibly it can be produced by relativistic electrons with the differential energy spectrum $dN(E) \propto dE/E$.

Nonstationary line-emission (moustaches) originates in very small ($\sim 0.5''$) short-lived (1^m - 20^m) grains at different depths ($0 \le \tau \le 0.4$). Spectroscopic investigations have shown that these moustaches are explosive or outburst-like phenomena. The atoms are ejected out of a small grain in two opposite directions along the magnetic lines of force with velocities up to 1000-2000 km/sec. The wings in great flares (magnitude ≥ 2) are composed of a cluster of moustaches of different widths.

The spectroscopic examination of the hydrogen spectrum of flares has shown that the population is nearly stationary only for the lower quantum levels ($n \le 3$). The electron gas in flares cools very rapidly (fraction of a second) as a result of inelastic collisions of electrons with H atoms, the main process being collisional excitation and ionization from upper levels. There should exist an energy source generating energy at the rate of ~ 500 ergs/cm^3·sec to counterbalance these losses during the lifetime of flare. It was shown that the mechanisms of electromagnetic heating and the penetration of coronal matter into the flare cannot provide the necessary amount of energy. Some experimental data are presented in favor of the importance of high-energy particles in the processes considered. The D_α-line strengthening above flares and moustaches indicates that possibly neutrons are also formed in these formations.

The acceleration mechanism can hardly be a statistical one (i.e., the Fermi mechanism). There is some evidence that not only weak but also strong magnetic fields on the sun (sunspots, etc.) possess a fine structure, and possibly the fields of separate field-tubes are as high as $\sim 10^4$ gauss. This offers the possibility of considering some kind of cyclotron acceleration of particles. The possibility of a "reversed" pinch-effect is also mentioned.

1. The Fine Structure of Emission from Active Formations

The results of the investigations of nonstationary solar processes carried out in the last four years are summed up in this paper.

With a high spectral resolving power (400-600 thousand), high dispersion (4mm-5mm/1A), and a very good quality of the photographs, the emission from active regions, as was originally discovered by us in 1954 [1], has a clearly marked fine structure. In regions of flares, growing sunspots and faculae the continuous and line emission is concentrated in short-lived (from 1.5^m to 30^m) "grains" with diameters of several hundred kilometers. Particularly remarkable and interesting is the transitory appearance (average lifetime — several minutes) of extremely fine (~ 0.5") luminous wings — "moustaches", on either side or only on one side of the unperturbed line, which sometimes extend up to 10-15 A from its center. These moustaches practically always have as their continuation an equally narrow strip of continuous emission. Examples of these phenomena are shown in Figs. 1 and 2. This peculiar continuous and line emission occurs over the whole of the disc close to or between small growing sunspots, in regions where faculae are observed and the tufty structure of the H_α line and outbursts are strongly marked. Recent detailed quantitative investigations [2] have led to the following results.

1. The continuous-emission strips differ from those produced by bright facular granules by their great brightness; generally speaking, they do not correspond to the positions of the strongest emission in the H_α, H and K lines (i.e., the locations of the calcium and hydrogen flocculi). They occur frequently in flares and close to small growing sunspots. The Fraunhofer lines are sometimes shifted towards the violet (by up to 5 km/sec) above the continuous-emission strips or have a Z-shaped appearance (as in the case of sunspots); if the shift is imperceptible, the line appears as if "flooded" by this continuous emission. The lifetime of this emission varies from several minutes (flares and moustaches) to $\sim 30^m$, although we cannot be certain that we are not observing the recurring appearance of continuous emission from almost the same position. The continuous emission is observed all over the solar disc and is localized in grains and not radial streams, since no increase of the strip widths (which are usually between 0.4-3") from the center to the limb is observed. (The width of the strips above the edge of the disc is as small (~1-2") as that near the center of the disc; see below.)

Direct measurements of the contrast of the strips in the region of the continuous spectrum and measurements of the line "flooding" (the measurement of the residual intensity of the lines over the strip) lead to the same results, which would be the case for optically thin emission (reference [2], § 3). This is also indicated by the fact that the intensity of the continuous-emission strips above the edge of the disc is only a few percent of the intensity of the disc (Fig. 2, a). It is also obvious that if the usual faculae were optically thick, but hotter formations, then the maximum intensity would be observed near the center and not very close to the edge of the disc (reference [2], § 3). The continuous-emission strips near the strong lines (such as H_α, H and K) lose their contrast and sometimes disappear in the region of the lines, in most cases at a distance of $\Delta\lambda \approx 0.8$ A (from the center of H_α); this leads to an average depth of occurrence $\tau = 0.4$ (in general, $0 \leq \tau \leq 0.6$; a similar result was recently obtained from H and K lines [3]). The presence of "flooding" or, in its absence, the occurrence of marked shifts reaching a value of 3 km/sec at the center of the disc, the appearance of continuous-emission strips in the chromosphere and above the limb, and the loss of continuous-emission contrast at different distances from the H_α, H and K lines provide evidence of: 1) the outflow of the continuous-emission grains, and 2) the different depths of their occurrence.

Thus, the continuous emission occurs in optically thin grains which rise from the lower layers of the atmosphere to various heights. The intensity of the actual continuous emission from these grains (corrected for the absorption $e^{-\tau \sec\theta}$ in the outer layers) increases smoothly towards the ultraviolet: its distribution is similar to that observed in AO stars up to $\lambda \approx 4000$ A, but near $\lambda = 3800$ A there is an indication of a maximum which shows that the continuous emission observed by us is similar to that in the spectra of RV Tau stars. There is no observable emission on either side of the Balmer series limit. The total flux of the continuous emission H is one-quarter of the solar radiation flux. If the flux were due to thermal radiation ($H \propto T^4$), then it would correspond to a temperature of $T \approx 4000°$, which contradicts the observed intensity distribution in the continuous-emission strips. Since the continuous-emission grains are optically thin, the emission per unit volume H/l, with $l = 500$ km, will be ~ 400 ergs/cm$^3 \cdot$sec. It is interesting that this figure is close to the energy produced in thermonuclear reactions inside the sun. It is obvious that this amount of energy cannot be produced by the redistribution of the photospheric thermal and turbulent energy, whose density does not exceed 10^3 ergs/cm$^3 \cdot$sec at the depths we are considering, since the energy source required for maintaining the production of the continuous emission during its lifetime (~100 sec) must be at least 100 times as efficient. These considerations are equally applicable to the continuous-emission strips in flares, as we have recently shown [4].

Recombination processes in an optically thin mass of hydrogen lead to an intensity distribution of the continuous emission [4,5] which is in complete disagreement with that observed (increased contrast in the red region)

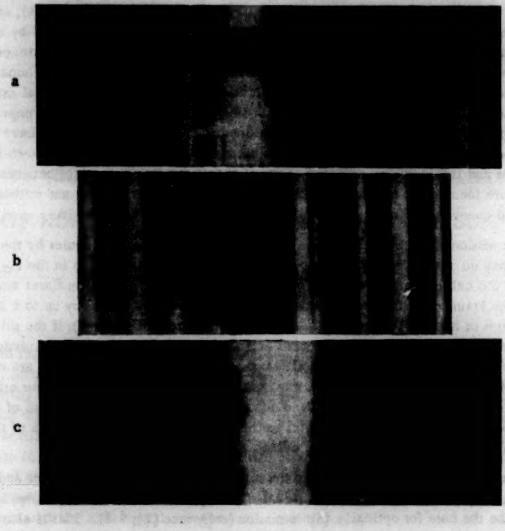

Fig. 1. Examples of nonstationary fine-structure emission (negative): a) in a flare (H_α line); b) strip of continuous emission near an active spot; c) typical "moustaches" on the H_α line.

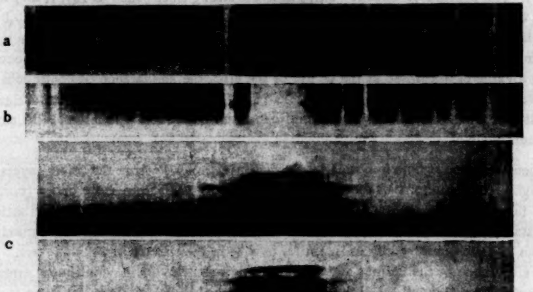

Fig. 2. a) A strip of continuous emission in the chromosphere (H_α line); b) moustaches on the H_α line inclined to the dispersion plane at the edge of the solar disc; c) and d) examples of moustaches above the edge of the solar disc (negative).

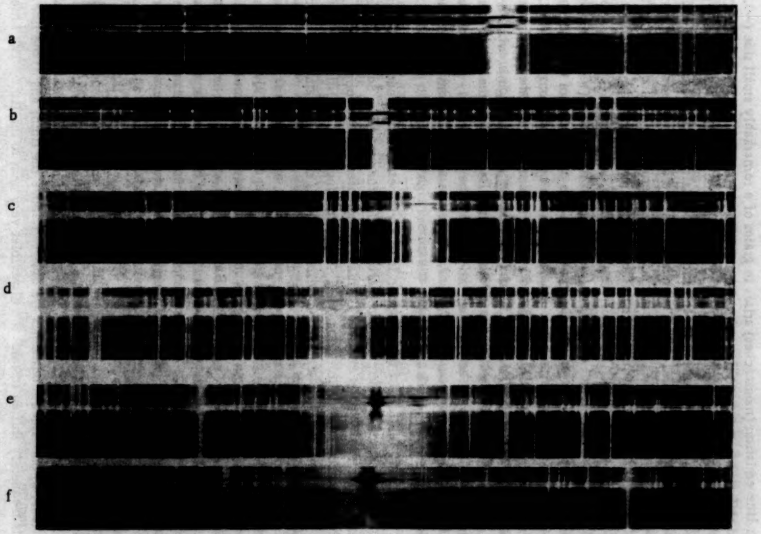

Fig. 3. The emission from a chromospheric flare in various spectral regions (negatives): a) H_α; b) H_β; c) H_γ; d) $H_\delta\,\delta$; e) H, H_ϵ; f) K.

and must therefore be rejected. The interpretation of the continuous emission in terms of scattering on possible electron condensations of the coronal type is also excluded, since in this case the intensity would remain constant over the spectrum; moreover, this mechanism requires an excessively high electron density in the condensations ($\sim 10^{18}$ cm^{-3}).

A reasonable agreement with the observed intensity distribution of the continuous emission is obtained on the assumption that the emission is produced by a cloud of relativistic electrons with an energy spectrum $dN(E) \propto dE/E$. In favor of this is the evidence from polarization measurements: in the continuous-emission strips the polarization differs from that in the directly adjacent spectrum of the unperturbed photosphere by 5-8%. Our recent measurements have confirmed this result which for individual emission grains is outside the limits of experimental error.

2. The peculiar line emission (moustaches) arises in grains of a remarkably small size ($\sim 0.4'' = 300$ km) and is briefly observed (lifetime: 1.5 to 20^m) on all of the Balmer series lines (H_α to H_{10}), the Ca$^+$, H, K, and ultraviolet (3706, 3737 A) lines, the Mg triplet (5167, 5173, 5184), some iron lines, and others; it has never been observed on the D_1 and D_2 sodium and He lines. On the strong lines (Balmer series) and the H and K lines, the moustaches can sometimes be traced for 10-15 A. The moustaches are also observed in the chromosphere above the solar limb, over faculae, and at the bases of electromagnetic and eruptive prominences (Fig. 2, c and d). As in the case of continuous emission, the emission moustaches sometimes "flood" the weak lines of the solar spectrum, while in the absence of "flooding" the lines are shifted towards the violet (by up to 3-5 km/sec). In some cases there are shifts towards the violet of the lines producing the moustaches (with a velocity of up to 8 km/sec) and these shifts are observed not only at the center, but also at the edge of the disc. In most cases the emission of moustaches stops or weakens at some distance from the center of strong lines (for H_α, on the average, at a distance of $\Delta\lambda \approx 0.75$ A; see Fig. 1, c), from which we can estimate the maximum depth at which the moustache emission occurs as $\bar{\tau}_{max} \simeq 0.4$. (The values $\bar{\tau} > 0.4$ are already inconsistent with the measured absolute intensities of the moustaches above the edge of the disc: if $\bar{\tau} > 0.4$, then the actual emission of the moustaches, corrected for the influence of the factor $e^{-\bar{\tau}\sec\theta}$, would be considerably greater than that measured above the limb.) All this shows that the moustache "grains," as well as the continuous emission associated with them, rise from the deep atmospheric layers and radiate at various heights.

One of the most interesting features of this emission is its asymmetry: independently of position on the disc, the violet wing of the moustaches is brighter and more extended than the red. Sometimes practically only the violet (less frequently — the red) wing is observed. This fact indicates that the formations are highly nonstationary or that the phenomenon is of the ejection type. The narrowing of the emission strip with distance from the center of the line shows that the phenomenon of moustaches is similar in nature to chromospheric streamers or eruptive prominences (Fig. 1, c and Fig. 2, b-d).

The spectroscopic investigation of the profiles near 30 moustaches has shown [2] that the width and the brightness of this emission in the Balmer lines are either constant or decrease very slowly with the number. In this respect the moustache emission is different from the flare emission, where H_α is the brightest and widest line, H_β is the narrowest and the least bright of the lines, while starting from H_γ the width and brightness are either constant or increase slightly up to H_8 and H_9. Neither damping nor the Stark effect can be used to explain this behavior of the moustache Balmer emission [2] except, possibly, the features of the extreme edges of the wings. The same width of the moustaches on the hydrogen and metal lines is also evidence against the widening effect of similar factors. The most probable explanation of the width of the moustaches is that it is due to the effect of macroscopic motions (similar to turbulence). The asymmetry could arise as the superposition of two effects: the ascent of the grain-producing continuous emission and a line absorption of this emission by the overlying layer. However, this explanation is unsatisfactory because 1) such an effect would disappear at the limb, which disagrees with observations; 2) one-sided moustaches are observed above the limb; and 3) the line, near which the moustaches arise, is frequently shifted towards the ultraviolet very near to the limb. The latter fact, the appearance of an asymmetry (irrespective of position on the disc), and the appearance of one-sided moustaches (particularly above the limb) definitely show that the phenomenon has the nature of an explosion or an ejection of atoms and is confined to a small region. The particles receding from the observer will always move in the direction of increasing optical depths and will be retarded more than the particles moving towards the observer. The narrowing of the moustaches away from the center of a line is characteristic of thin ejections and streamers. The inclination of the moustaches to the dispersion plane, observed in a number of cases (for example, in Fig. 2, b), as well as the one-sided moustaches, are evidence for the interpretation of this phenomenon as an explosion or an ejection which takes place mainly in two opposite directions. Sometimes one of the moustache wings is situated nearer to (or farther from) a small sunspot than the other wing, while in a number of cases it could be determined with certainty that the moustache streamers lay along the force lines of the sunspot field. The appearance of a sudden widening simultaneously with an increase in brightness is a feature not only of flares ([6], see below), but also of knots in the eruptive prominences [7] in which, according to measurements [8], this phenomenon occurs in the form of streamers ejected in opposite directions with supersonic velocities. This indicates the presence of a magnetic field which is transported together with the knots and which restricts the motion of the plasma to two opposite directions.

In connection with this phenomenon, the occurrence of short-lived accelerations, greater than those due to gravity, in eruptive and in dark and bright (above the limb) streamers formed near sunspots [8,9] attracts attention. The occurrence of such accelerations is very puzzling unless we interpret them as the result of some type of "explosion."

The observed asymmetry of the moustaches, which was explained as being due to the motion of the atoms, leads to velocities of the order of several hundred km/sec, but if the moustaches are in the form of streamers, then the atomic velocities can be as high as 1000 km/sec or more. Thus, if the moustaches are the sources of corpuscular emission, these sources must be localized in very small volumes and the ejection of corpuscles from the solar atmosphere must be a sporadic phenomenon. This is confirmed by the discontinuous nature of the disturbances in the terrestrial magnetic field and, particularly, by the disturbances which have a sudden onset. It is also significant that, apparently, these disturbances are not associated with any process of radial outflow.

2. Flares

The highly nonstationary nature of flares was first indicated by the following results of the investigation of their development by means of H_α motion pictures [6]: 1) the H_α-intensity of the flares increases simultaneously with the increase of the area occupied by the flare; 2) the lifetime of the flares increases with their area, which is characteristic of all dissipative relaxation processes; 3) the faster a flare develops, the greater is the maximum brightness reached — a feature which is characteristic of nova explosions and unstable stars. Further, in a number of cases fast moving wavefronts of light-emitting matter are observed in flares; the velocities observed are supersonic and can be as high as 300 km/sec [6]. The width of the light-emitting front (or transition layer) decreases with increasing velocity, as was shown by recent measurements [7], in the same way as the width of a shock-wave front. Measurements have shown that these wave fronts bend around sunspots or partially cover the sunspots; encountering the strong sunspot fields, the wave front is sometimes retarded completely and comes to rest increasing in brightness and then slowly fades. It is possible that this situation is responsible for the appearance of the peculiar moustache emission directly at the edge of a sunspot: when a shock wave, propagating from the "explosion," encounters the sunspot field, it is reflected and spreads along the lines of force in opposite directions (mainly towards and away from the observer — depending on the topography of the field). It is possible that, because of this, one of the moustache wings is sometimes situated closer to the sunspot than the other. Investigations reported in reference [10] have shown that the appearance of ejections and streamers near sunspots is accompanied by radio bursts, i.e., a type of nonlinear process which is characteristic of shock waves in a plasma. It is in regions with plasma motions in opposite directions that conditions arise favorable for the escape of radio radiation through the Doppler shift of the emitted frequency relative to the critical frequency, as was shown in references [11].

The investigation of the fine structure of the emission (§1) has shown that the emission is also localized in small short-lived grains of a somewhat larger size (~1000 km)(see Figs. 1 and 3); however, this emission mainly covers the centers of the lines (Fig. 3) and changes with the number of the Balmer lines differently from the moustache emission. In good photographs, we have recently found that the very wide wings observed in big flares (of magnitude ≥ 2) consist of a closely crowded series of moustaches of various lengths whose position, in general, does not coincide with that of the emission strips in the body of the H_α-line and which, properly speaking, are associated with what may be called chromospheric flares. Because of the image unsteadiness, these wide wings are most frequently not resolved in the photographs into separate moustaches, but it is clearly noticeable that the wings are of different extent in different locations of the flare and are widest in the region closest to a sunspot.

Similar investigations of the hydrogen emission spectrum of flares, carried out recently [12] at high dispersion and resolving power (Fig. 3 gives an example of the spectrum), have shown that all of the hydrogen lines (including H_α) are widened because of the Stark effect if it is assumed that $N_2 = 10^{15}$-10^{16}, p_e = 25-50 bars, and $T_{kin} = 10^4$ °K.

The Balmer decrement of this emission is very different from that for the case of recombination. The investigation of all the possible mechanisms for hydrogen excitation and ionization in a flare [4] shows that: 1) the population density of the lower levels ($n \leq 3$-4) is practically stationary and is governed by the flare radiation field; 2) the occupation of the higher levels ($n > 4$) is governed by the processes of excitation and ionization by electron collisions; 3) the number of atoms in the ground state in the flares is $\approx 10^{13}$ cm^{-3}. As the result of the collisions of electrons with hydrogen atoms, the electron gas in a flare cools very rapidly (in a time which is negligibly small by comparison with the lifetime of the flare) unless energy is supplied it by an energy source whose power output is ≈500-700 ergs/cm^3·sec. It is important to note that this value is practically the same as the amount of energy liberated in the continuous-emission grains of both flares and moustaches. It is easily found that the mechanism by which the plasma is heated by hydromagnetic waves cannot provide this amount of energy even if the additional frictional losses, associated with incomplete ionization [13], are taken into account.

The hypothesis that hot coronal matter penetrates into the region of the flare is also inadequate for the explanation of this energy liberation. The appearance in the corona above the flares of the yellow emission line CaXV (if the identification is correct) with an ionization potential of 814 ev indicates that the temperature above the flare is ~10^6 °K. If this matter penetrates into the flare, then the stored energy per cm^3, which is equal to ~10^{-9} n_e, will be adequate for providing energy at the rate of 500 ergs/cm^3·sec for ~100 sec, only if $n_e > 10^{14}$. However, this value of the electron density n_e is not only inconsistent with the density of the corona, but even with electron density in the flare. Moreover, the emission of the yellow line continues long after the disappearance of the flare.

According to recent rocket observations, flares emit hard x-radiation with wavelengths < 1 A; its flux is ~10^{-4} ergs/cm^2·sec at the top of the earth's atmosphere. If the size of the region occupied by the flares is ~1% of the solar radius, then the density of the γ-quanta (if they are present in this radiation) in the flare will be of the order of 1 cm^{-3}. (An allowance for the absorption of the γ-quanta by the upper layers is unlikely to increase this estimate to more than 100 cm^{-3}.) If this radiation is due to the synchrotron emission by electrons, then the electron density will be

$$n_e < 10^2/n_i c\sigma = 10^5\,\mathrm{cm}^{-3},$$

on the assumption that the number of protons $n_i \approx 10^{13}$ and $\sigma \approx 0.5 \cdot 10^{-26}$. Thus the presence of x-radiation from flares indicates that electrons (possibly relativistic) may occur in flares with concentrations ~10^5 cm^{-3}. The radio spectra of flares also indicate the presence of beams of particles with velocities of 50,000-100,000 km/sec, which in the case of protons corresponds to energies of ~10^8 ev [14]. The sudden increases in the cosmic ray flux during flares also indicates the appearance of particles with an energy of ~10^9 ev; the flux of the flare cosmic rays is ~1 particle/cm^2·sec (at the top of the earth's atmosphere) [15]. Without considering at the moment the problem of the production of these particles, we can estimate the relativistic-particle density which is required for supplying energy at the rate of ~500 ergs/cm^3·sec to the electron gas of the flare. In hydrogen with a density of $n \approx 10^{13}$ atoms·cm^{-3}, the energy losses of the relativistic electrons of energy ~10^9 ev are given by (for example, see reference [16])

$$\left(\frac{dE}{dt}\right)_{\mathrm{ion}} = 1\cdot 2\cdot 10^{-20}\, n \left\{20.1 + 3\ln\frac{E}{mc^2}\right\} \approx 10^{-5}\ \mathrm{erg/sec};$$

$$\left(\frac{dE}{dt}\right)_{\mathrm{mag}} \approx 1.60\cdot 10^{-15} H^2 \left(\frac{E}{mc^2}\right)^2 \approx 10^{-5}\ \mathrm{erg/sec};$$

$$\left(\frac{dE}{dt}\right)_{\mathrm{rad}} \approx 8.0\cdot 10^{-16}\, nE \approx 5\cdot 10^{-5}\ \mathrm{erg/sec};$$

on the assumption that H = 100 gauss, in accordance with the measured magnetic fields in the region of flares. Thus, if the energy of the electron gas is maintained as the result of the energy losses of the relativistic particles, then the density of such particles will be determined by the condition $5 \cdot 10^{-5}\, n_{rel} \approx 500$ or $n_{rel} \approx 10^7$ cm^{-3}, i.e., it will be of the order of 10^{-6} per cent of the density of thermal electrons in a flare, $n_e \approx 10^{13}$). A similar estimate is obtained in the case of protons. The flux of such particles at the earth's surface $\approx cSn_{rel}/r^2$ (where S is the area covered by the flare, and r is the distance of the earth from the sun) and is unlikely to be greater than 100, since the size of the region in which these particles are produced must not exceed several kilometers. (If the field is $\sim 10^3$ gauss, this size will be ~ 100 km.) However, these conclusions cannot be regarded as being certain since the diffusion of the particles in the magnetic field between the sun and the earth has been neglected (see references [4] and [17]).

3. The Production of High-Energy Particles in the Solar Atmosphere

Let us summarize the facts which provide evidence for the production of streams of particles, including high-energy particles, in the active regions: a) the occurrence of bursts of cosmic radiation of solar origin during large flares; b) the emission by the flares (more likely by the accompanying moustaches) of hard x-radiation (possibly γ-rays); c) the production of particles with velocities of $(5-10) \cdot 10^4$ km/sec (according to the data on the spectra of large radio bursts); d) the correspondence between the spectrum of the continuous emission and that of the synchrotron radiation of relativistic particles with $\gamma \simeq 1$ and the polarization of the continuous emission; e) the energy of the electron gas in a flare can be maintained only if relativistic particles are present; f) the appearance of moustaches provides evidence that streams of atoms (with velocities as high as ~ 1500 km/sec) are produced, mainly in the photosphere, above flares as well as in the vicinity of the sunspot magnetic fields.

The problem of the generation of high-energy particles in the atmospheres of the sun and stars is the most difficult one in modern astrophysics. The well-known Fermi mechanism is associated with great difficulties. If the Fermi mechanism operates in an active region on the sun, then the increase in the particle energy

$$\frac{dE}{dt} = \frac{u^2}{c^2} \cdot \frac{v}{l} E$$

when it moves between gas turbulences (having a velocity u) must be greater than 10^{-5}. Assuming $v \approx c$ and $u \leq 50$ km/sec, we find $l \leq 10^5$ cm. At present, there are no experimental data which provide evidence for the existence of such small-scale formations in the solar atmosphere. Further, the distance through which the particles diffuse, $D \approx \sqrt{l\,vt}$, where t is the lifetime of the active formation ($\sim 10^3$ sec), will not be less than 10^{10} cm, so that these particles will leave the characteristic volume (of size 10^7-10^8 cm) in a fraction of a second. Because of these considerations, the assumption that the particles are accelerated to an energy of $\sim 10^9$ ev by a statistical mechanism is not plausible. We have recently shown [18] that in moustaches and flares, as well as in active sunspots, the deuterium line D_α is strengthened, which indicates that neutrons are produced in these regions. It is possible that in this case the neutrons are produced by the same mechanism as in the case of accelerators where protons of energy ~ 400 Mev are scattered by the neutrons of nuclei and a beam of neutrons of energy ~ 350 Mev is produced in the direction of the original proton beam (see reference [19]). In general, we can assume that either the original generation of the high-energy particles occurs in a nuclear explosion or nuclear reaction or, alternatively, that the nuclear processes, whose existence in stellar atmospheres has been proposed by a number of authors [20], are initiated by the high-energy particles generated by some unusual accelerating mechanism (not the Fermi mechanism). Finally, the nonstationary processes and the production of high-energy particles could be the result of the peculiar properties (instability) of a plasma in a magnetic field; such properties are similar to those recently investigated by L. A. Artsimovich in his experiments on the pinch effect [21].

As yet, there are no experimental grounds for considering that the nuclear explosions or reactions are the sources of high-energy particles on the sun (the energy liberated in known reactions is inadequate for acceleration) unless we introduce the idea of the transportation to the solar surface of unstable "protomatter" (supernuclei), whose properties are at present completely unknown [20].

The difficulties associated with a theory of the occurrence and operation of accelerating mechanisms in the presence of a magnetic field (for example, of sunspots) are well known; they arise because the magnetic field is "frozen into" the moving plasma and because there is a large depolarization effect as the result of collisions: for fields of 10^3 gauss and densities of $\sim 10^{-8}$ g/cm^3, the cyclotron frequency in the region of sunspots is less than or comparable with the collision frequency. However, recent investigations of the magnetic fields, carried out at the Crimean Astrophysical Observatory [22], have shown that there is a very fine structure, or microstructure, in the magnetic fields, not only in the regions free from sunspots where the fields vary over a distance of several seconds of arc, but also directly within the region of a sunspot. The wings of the σ-component of a magnetically-split line are fairly wide in sunspots and the π-component is almost always present (even in the case of sunspots at the center of the disc). Chevalier and Thissen [23] have reported that luminous granules appear in sunspots. Thus, it is quite likely that the magnetic field inside the sunspots has a fine structure, the smoothing out of which during observations leads to values $\sim 10^3$ gauss, while in fact the field in separate regions of the sunspot (possibly with different polarity and direction) is of the order of 10^4 gauss, i.e., the situation may be the same as in the case of granules where the smoothing out of the fields of individual granules (~ 100 gauss) during observations leads to fields of the order of several gauss. If this assumption is confirmed (as the result of further investigations) and the sunspot fields are as high as $\sim 50{,}000$ gauss, confined to small tubes, then the problem of particle acceleration can be solved fairly easily. In this case, the plasma in the region between the tubes will be highly diamagnetic and its ejection with high velocities is possible [24].

From a consideration of the energy involved, it can be seen that the sunspots cannot be produced at the expense of the energy in the outer layers of the sun (the energy density in a sunspot, even with a field of 3000 gauss, is $H^2/8\pi \approx 10^6$, while the thermal and turbulent energy of the photosphere does not exceed 10^4 with $\rho \approx \approx 10^{-8}$ and velocity $v > 10$ km/sec). Therefore, phenomenologically speaking, the sunspot energy is sufficient to maintain the nonstationary emission from an active region, even at a rate of 1000 ergs/cm^3·sec, during its observed lifetime of $\sim 10^3$ sec. The energy density in the streams near to sunspots (in moustaches) is even higher: $\frac{1}{2}\rho v^2 \approx 10^5$ ergs/cm^3 (on the assumption that the hydrogen density is the same as that in flares, $n \approx 10^{13}$ cm^{-3}, and that the velocities are $\approx 10^3$ km/sec), so that not only flares, but also moustaches can arise as the result of the conversion of the sunspot magnetic-field energy into the kinetic energy of the particles. Similarly, the observed wave fronts (shock waves) in flares, moving with velocities of ~ 300 km/sec, can arise through the conversion of sunspot magnetic energy into the energy of the shock waves.

We also should not overestimate the difficulties associated with the freezing-in of the magnetic field, which prevents an appreciable flow of current in a region occupied by the magnetic field (motion at right angles to the field), particularly in view of the fine structure of the nonstationary formations on the sun. It is probable that phenomena of the type of an "inverse" pinch effect occur – the conversion of magnetic energy into a current in the presence of neutral points and regions. These regions, as we have recently found (see also reference [25]), are the location of flares. The solar plasma will be compressed in the vicinity of a neutral region if initially the magnetic-energy density is greater than the thermal-energy density. Calculations show that this compression, accompanied by the conversion of magnetic into thermal energy, continues until there is a region with dimensions of $\sim 10^4$ cm in which a very large current is produced and the temperature rises to several million degrees ("inverse" pinch effect). The time required for this process (the time required for a hydromagnetic wave to propagate through the characteristic distance – 10^8 cm) is only about 10 sec. The behavior of plasma near neutral points is also discussed in reference [26].

It is obvious that these problems are of importance not only in astrophysics, but also in the physics of controlled thermonuclear reactions and particularly in connection with the problem of producing and maintaining large currents in a plasma under the influence of a magnetic field.

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THE DIFFUSION OF L_α - RADIATION IN NEBULAE AND STELLAR ENVELOPES

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The diffusion of L_α-radiation in a medium with an arbitrary velocity gradient and any contour of the absorption coefficient is considered. It is supposed that the radiation diffuses with complete redistribution in frequency. For a one-dimensional medium the problem is reduced to the determination of the function $B(\tau)$ from the integral equation (8). An approximate solution of this equation is given by Formula (11). The derived solution is generalized to a three-dimensional medium, composed of plane-parallel layers. An expression for the pressure due to L_α-radiation has been found.

Two limiting cases of the problem have been considered separately: I) the medium has a large velocity gradient (this case for a rectangular contour of the absorption coefficient was considered earlier by the author), II) the medium is not in motion (this case for a Doppler contour of the absorption coefficient was considered by Zanstra and others). In case I the expression for $B(\tau)$ does not depend on the contour of the absorption coefficient. However, the radiation pressure for various contours is different. For a Doppler contour the radiation pressure is several times as large as that for a rectangular contour.

The character of the field of L_α-radiation in a medium depends on the value of the contour parameter δ, given by Formula (57). If $\delta \gg 1$, the field of radiation in a medium is similar to that in case I; if $\delta \ll 1$ – that in case II. For planetary nebulae $\delta \simeq 1$, for envelopes of novae $\delta \gg 1$.

1. Introduction

The photoionizations of hydrogen atoms and the subsequent recombinations, which occur in gaseous nebulae, lead to the appearance of L_α-quanta. As a result of the large optical thickness of the nebulae in the Lyman lines, these quanta diffuse for a long time through the nebulae. Therefore, the density of L_α-quanta is found to be very high in the nebulae.

Analogous processes occur in the envelopes of novae, Be and WR stars, and other objects.

The problem of the diffusion of L_α-quanta is of interest for many reasons. In particular, it is of interest because the radiation pressure caused by the L_α-radiation plays an important role in the dynamics of nebulae and stellar envelopes.

V. A. Ambartsumian in 1933 [1] was the first to investigate the diffusion of L_α-quanta in nebulae. He assumed that the nebula is motionless or is expanding without a velocity gradient. Later, Zanstra [2] considered the same problem for a nebula expanding with a velocity gradient. However, in both these investigations it was assumed that the diffusion of L_α-quanta takes place without a change in frequency.

In 1944, the author [3] investigated the diffusion of L_α-quanta in a nebula, expanding with a velocity gradient, on the assumption of a complete redistribution of the radiation with frequency. This assumption means that the probability for an element of volume to emit a quantum of frequency ν does not depend on the frequency of the absorbed quantum and is proportional to the coefficient of absorption in the line σ_ν. In this paper it was assumed that the contour of the absorption coefficient is rectangular. When a velocity gradient is present in the nebula, some of the quanta escape from the central regions of the nebula as a result of the Doppler effect. In the case of a complete redistribution of radiation with frequency inside the line, the fraction of quanta escaping can be fairly large. Therefore, the values of the L_α-radiation density and of the radiation pressure produced by it were found to be considerably lower than those obtained earlier by Ambartsumian and Zanstra.

In 1949, Zanstra [4] investigated the L_α-radiation diffusion in a motionless nebula, likewise assuming a complete redistribution of the radiation with frequency. Having adopted the Doppler contour for the absorption coefficient, Zanstra noticed that in the case of a complete redistribution of the radiation with frequency inside the lines, some of the quanta can escape from the central regions of the nebula in the line wings, where the optical thickness of the nebula is small. He also obtained values of the L_α-radiation density and pressure considerably smaller than those found earlier.

Further investigations of the L_α-quantum diffusion in a motionless nebula on the assumption of a complete redistribution of the radiation with frequency were made by Miyamoto [5], Zanstra [6], Koelbloed [7], and others.

However, strictly speaking, the assumption of a complete redistribution of the radiation with frequency is not valid. The true law of the redistribution of radiation with frequency in the lines for the case of nebulae was discovered by Henyey [8]. The diffusion of L_α-quanta in a motionless nebula with the use of the correct frequency redistribution was investigated by Unno [9] and the author [10]. In this work various methods were used and numerical results were obtained for a number of cases. One of the general results of this work is that the values of the L_α-radiation density and pressure obtained using the correct frequency redistribution are not very different from the values of these quantities obtained on the assumption of complete redistribution with frequency. Therefore, the investigation of radiation diffusion for the case of complete redistribution with frequency can be adequate in many applications.

In the present paper, we consider the problem of the L_α-radiation diffusion with a complete redistribution with frequency, for the case of an arbitrary absorption-coefficient contour and an arbitrary velocity gradient in the medium. The general solution of the problem is obtained and from this follow the solutions for two limiting cases: I) the medium moves with a large velocity gradient (this case for a rectangular absorption-coefficient contour was investigated earlier by the author [3]), II) the medium is motionless (this case for a Doppler absorption-coefficient contour was studied by Zanstra [4] and others).

In the early sections of the paper the L_α-radiation diffusion in a one-dimensional medium is investigated; next, a three-dimensional medium consisting of plane-parallel layers is considered. An approximate expression for the source function $B(\tau)$ is found and the radiation pressure produced by the L_α-radiation is determined. Finally, the nature of the L_α-radiation field in planetary nebulae and nova envelopes is discussed.

2. One-Dimensional Medium

For a start, let us investigate the diffusion of radiation in a one-dimensional medium. Let s_0 denote the thickness of the medium and s the distance to one of its boundaries. We will assume that the medium is in motion and that the velocity v is a function of s.

We will represent the volume absorption coefficient at a frequency ν, for an observer moving with the given volume, by $\sigma_\nu(s) = \sigma_{\nu_0}(s)\alpha(x)$, where $x = (\nu - \nu_0)/\Delta\nu_D$, ν_0 is the central frequency of the line for the same observer, and $\Delta\nu_D$ is the Doppler halfwidth of the line. Instead of the distances s and s_0, let us introduce the optical depths

$$\tau = \int_0^s \sigma_{\nu_0}(s)\,ds, \quad \tau_0 = \int_0^{s_0} \sigma_{\nu_0}(s)\,ds. \tag{1}$$

The quantity τ_0 represents the optical thickness of the medium when $dv/ds = 0$. In the presence of a velocity gradient in the medium, τ_0 is a fictitious optical thickness corresponding to the central values of the coefficient of absorption in the different layers of the medium.

Let $\epsilon(\tau, x)d\tau dx$ be the amount of energy radiated in one direction per second at a depth between τ and $\tau + d\tau$ in the frequency interval dx at x. With the assumption of a complete redistribution of the radiation with frequency, the quantity $\epsilon(\tau, x)$ can be written in the form:

$$\varepsilon(\tau,\ x) = \alpha(x) B(\tau). \tag{2}$$

Let $I_1(\tau, x)$ and $I_2(\tau, x)$ denote the intensities of the radiation, with frequency x, moving at a depth τ in the direction of increasing and decreasing thickness, respectively, [for an observer moving with a velocity $v(\tau)$]. In order to express $I_1(\tau, x)$ and $I_2(\tau, x)$ in terms of the quantity $B(\tau)$, we must take into account the fact that the radiation, which has moved from a depth τ' to the depth τ, experiences a frequency shift by an amount $\frac{\nu_0}{c\Delta\nu_D}[v(\tau') - v(\tau)]$. Assuming that $dv/d\tau = \text{const.}$ and $dv/d\tau \geq 0$, we find that this shift is by $-\gamma\,|\,\tau' - \tau\,|$, where

$$\gamma = \frac{1}{u}\frac{dv}{d\tau}, \tag{3}$$

in which u is the mean thermal velocity of the atoms $\left(\Delta\nu_D = \nu_0 \frac{u}{c}\right)$.

From the above considerations, we find that the intensities are given by

$$I_1(\tau,\ x) = \int_0^{\tau} B(\tau')\,\alpha\,[x + \gamma(\tau - \tau')]\, e^{-\int_0^{\tau-\tau'} \alpha(x+\gamma z)dz}\, d\tau', \tag{4}$$

$$I_2(\tau,\ x) = \int_{\tau}^{\tau_0} B(\tau')\,\alpha\,[x + \gamma(\tau' - \tau)]\, e^{-\int_0^{\tau'-\tau} \alpha(x+\gamma z)dz}\, d\tau'. \tag{5}$$

Let $\epsilon_0(\tau, x) = \alpha(x)B_0(\tau)$ denote the emission coefficient which is directly determined by the radiation sources located in the medium (in the case of nebulae and stellar envelopes it is determined by the conversion of L_c-quanta into L_α-quanta). Assuming that the diffusion is not accompanied by true absorption, we obtain

$$B(\tau) = \frac{A}{2}\int_{-\infty}^{+\infty} [I_1(\tau,\ x) + I_2(\tau,\ x)]\,\alpha(x)\,dx + B_0(\tau), \tag{6}$$

where

$$A\int_{-\infty}^{+\infty} \alpha(x)\,dx = 1. \tag{7}$$

The substitution of (4) and (5) into (6) yields

$$B(\tau) = \frac{1}{2}\int_0^{\tau_0} B(\tau')\,K(|\tau - \tau'|)\,d\tau' + B_0(\tau), \tag{8}$$

where

$$K(t) = A\int_{-\infty}^{+\infty} \alpha(x + \gamma t)\, e^{-\int_0^{t} \alpha(x+\gamma z)dz}\,\alpha(x)\,dx. \tag{9}$$

The integral Eq. (8) is the basic equation for the problem being considered.

3. Approximate Solution

In order to obtain an approximate solution of the integral Eq. (8), we make use of some considerations concerning the physics of the radiation diffusion with a frequency redistribution. In the present case, after each elementary act of scattering, irrespective of its location in the medium, the quanta produced can escape from the medium directly from this point. When the medium is motionless, quanta can escape from any depth τ if they are emitted in the wings of a line, i.e., if their frequencies satisfy one of the following inequalities: $\alpha(x)\tau \ll 1$ or $\alpha(x)(\tau_0 - \tau) \ll 1$. If the medium moves and there is a velocity gradient, then the number of quanta produced at a given point and which escape from the medium will be even larger because of the Doppler effect.

Roughly speaking, each quantum, produced through the action of the radiation sources at any point in the medium, will escape after diffusing through a comparatively small region. Therefore, we can consider that, approximately, the number of quanta produced in any region is equal to the number of quanta leaving the medium from this region.

An approximate expression for the function $B(\tau)$ can be easily obtained with the help of the considerations given above. In this way, the author in a previous paper [3] calculated the function $B(\tau)$ for a medium moving with a large velocity gradient when the absorption coefficient has a rectangular contour.

Let us use this method for solving Eq. (8). Putting the function $B(\tau')$ equal to $B(\tau)$, and taking it out of the integral, we obtain approximately

$$B(\tau)\left[1 - \frac{1}{2}\int_0^{\tau} K(\tau)\,d\tau - \frac{1}{2}\int_0^{\tau_0-\tau} K(\tau)\,d\tau\right] = B_0(\tau). \tag{10}$$

From this it follows that

$$\frac{1}{2}\left[L(\tau, \gamma) + L(\tau_0 - \tau, \gamma)\right] B(\tau) = B_0(\tau), \tag{11}$$

where

$$L(\tau,\gamma) = A\int_{-\infty}^{+\infty} \alpha(x)\, e^{-\int_0^{\tau} \alpha(x+\gamma z)\,dz}\, dx = A\int_{-\infty}^{+\infty} \alpha(x) \exp\left\{-\frac{1}{\gamma}\int_x^{x+\gamma\tau} \alpha(y)\,dy\right\} dx. \tag{12}$$

It is obvious that the quantity $^1/_2\,[L(\tau,\gamma) + L(\tau_0 - \tau, \gamma)]$ represents the fraction of quanta which leave the medium from a depth τ. Eq. (11), in accordance with what was said above, is a description of the fact that the number of quanta escaping from a given region is equal to the number of quanta produced in this region.

Of course, Eq. (11) can be used only for the middle (i.e., sufficiently distant from the boundaries) layers of the medium and also when the function $B_0(\tau)$ does not vary rapidly.

In stellar envelopes and nebulae, the value of τ_0 is usually very large, while the value of γ is usually very small (see § 7). Therefore, two limiting cases of the problem under consideration are of interest:

I) The velocity gradient in the medium is large, as a result of which $\gamma\tau \gg 1$. We will assume that $\tau = \infty$. In this case, the quanta escape from the medium only through the Doppler effect.

II) The velocity gradient in the medium is small, so that $\gamma\tau \ll 1$. We will take $\gamma = 0$. In this case the quanta escape from the medium only in the wings of the lines.

For case I, we obtain from (12):

$$L(\infty, \gamma) = A\int_{-\infty}^{+\infty} \alpha(x)\, e^{-\frac{1}{\gamma}\int_x^{\infty} \alpha(y)\,dy}\, dx = A\gamma\left(1 - e^{-1/A\gamma}\right). \tag{13}$$

For $A\gamma \ll 1$, we find that

$$L(\infty,\ \gamma) = A\gamma. \tag{14}$$

It must be noted that the quantity $A\gamma$ is independent of the shape of the absorption coefficient contour. In fact, we have

$$\int \sigma_\nu d\nu = \frac{h\nu_0}{c} n_1 B_{12}, \tag{15}$$

where n_1 is the number of atoms per cm^3 occupying the first state, B_{12} is the Einstein absorption coefficient. Substituting $\sigma\nu = \sigma\nu_0\ \alpha\ (x)$ into (15), we get

$$\sigma_{\nu_0} \frac{\Delta\nu_D}{A} = \frac{h\nu_0}{c} n_1 B_{12}. \tag{16}$$

With the help of (3) and (16), we find

$$A\gamma = \frac{1}{hn_1 B_{12}} \cdot \frac{dv}{ds}. \tag{17}$$

Thus, in the case considered, the number of quanta which escape from the medium is independent of both $\alpha(x)$ and τ.

For case II, Eq. (12) yields

$$L(\tau,\ 0) = A \int_{-\infty}^{+\infty} \alpha(x)\, e^{-\tau\alpha(x)}\, dx. \tag{18}$$

The number of quanta which escape from the medium now depends strongly on $\alpha\ (x)$ and τ.

Let us take as an example

$$\alpha(x) = e^{-x^2}. \tag{19}$$

Instead of (18), we now have

$$L(\tau,\ 0) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-x^2 - \tau e^{-x^2}}\, dx. \tag{20}$$

For $\tau \gg 1$, Eq. (20) yields the following asymptotic expression:

$$L(\tau,\ 0) = \frac{1}{\tau\sqrt{\pi \ln\tau}}. \tag{21}$$

It can be assumed that the accuracy of Eq. (11) increases with increasing velocity gradient in the medium. However, even for $dv/d\tau = 0$, the use of Eq. (11) does not involve a very large error (see § 6).

4. Radiation Pressure

Knowing the function $B(\tau)$, we can determine the magnitude of the pressure exerted by the Lα-radiation. Let $P(\tau)$ denote the magnitude of the radiation pressure acting on a unit volume at a depth τ. For a one-dimensional medium

$$P(\tau) = \frac{\sigma_{\nu_0}}{c} \int_{-\infty}^{+\infty} [I_1(\tau,\ x) - I_2(\tau,\ x)]\, \alpha(x)\, dx. \tag{22}$$

Substituting expressions (4) and (5) into (22), we get

$$P(\tau) = \frac{\sigma_{\nu_0}}{c} \int_{-\infty}^{+\infty} \alpha(x)\,dx \left\{ \int_0^{\tau} B(\tau')\,\alpha[x+\gamma(\tau-\tau')]\,e^{-\int_0^{\tau-\tau'} \alpha(x+\gamma z)dz}\,d\tau' - \right. \tag{23}$$

$$\left. - \int_{\tau}^{\tau_0} B(\tau')\,\alpha[x+\gamma(\tau'-\tau)]\,e^{-\int_0^{\tau'-\tau} \alpha(x+\gamma z)dz}\,d\tau' \right\}$$

Let us investigate the case of a medium moving with a large velocity gradient. In this case, in the middle layers of the medium $B(\tau) = B_0(\tau)/A\gamma$ (for $A\gamma \ll 1$). We will assume that $B_0(\tau)$ is a slowly and monotonically increasing function. Then, for calculating $P(\tau)$ the function $B(\tau')$ can be written in the form

$$B(\tau') = B(\tau) + \frac{dB}{d\tau}(\tau'-\tau). \tag{24}$$

Substituting (24) into (23), we find that in this case the term which contains $B(\tau)$ can be neglected. Therefore, we have

$$P(\tau) = -\frac{\sigma_{\nu_0}}{cA}\frac{dB}{d\tau}[\Phi(\tau,\gamma) + \Phi(\tau_0-\tau,\gamma)], \tag{25}$$

where

$$\Phi(\tau,\gamma) = A\int_{-\infty}^{+\infty} \alpha(x)\,dx \int_0^{\tau} t\alpha(x+\gamma t)\,e^{-\int_0^t \alpha(x+\gamma z)dz}\,dt =$$

$$= -\tau L(\tau,\gamma) + \int_0^{\tau} L(t,\gamma)\,dt. \tag{26}$$

It is easy to find that

$$\int_0^{\tau} L(t,\gamma)\,dt = A\int_{-\infty}^{+\infty} [1 - e^{-\frac{1}{\gamma}\int_x^{x+\gamma\tau} \alpha(y)dy}]\,dx. \tag{27}$$

Substituting (12) and (27) into (26), we find

$$\Phi(\tau,\gamma) = A\int_{-\infty}^{+\infty} \left\{ 1 - [1 + \alpha(x)\tau]e^{-\frac{1}{\gamma}\int_x^{x+\gamma\tau} \alpha(y)dy} \right\} dx. \tag{28}$$

Eq. (27) can also be written in the form

$$\int_0^{\tau} L(t,\gamma)\,dt = 2A\int_0^{\infty} \left[1 - e^{-\frac{1}{\gamma}\int_x^{x+\gamma\tau} \alpha(y)dy}\right] dx + A\int_0^{\gamma\tau} \left[1 - e^{-\frac{1}{\gamma}\int_{x-\tau\gamma}^{x} \alpha(y)dy}\right] dx. \tag{29}$$

For $\gamma\tau \gg 1$ and $\gamma \ll 1$, the last term in (29) can be replaced by $A\gamma\tau$. Therefore, for $\Phi(\tau,\gamma)$ we get

$$\Phi(\tau,\gamma) = 2A\int_0^{\infty} \left[1 - e^{-\frac{1}{\gamma}\int_x^{x+\gamma\tau} \alpha(y)dy}\right] dx + A\gamma\tau - \tau L(\tau,\gamma). \tag{30}$$

In order to calculate $\Phi(\tau,\gamma)$, we must assign some form to the contour of the absorption coefficient. We will assume that the absorption coefficient is determined by the thermal motions of atoms, i.e., $\alpha(x)$ is given by Eq. (19). Taking into account that with the adopted absorption coefficient the quantity $L(\tau,\gamma)$ for large values of $\gamma\tau$ is given to a high accuracy by Eq. (14), we find from (30) that

$$\Phi(\tau,\ \gamma) = \frac{2}{\sqrt{\pi}} x_1, \tag{31}$$

where x_1 is given by the relation

$$\int_{x_1}^{\infty} e^{-x^2}\, dx = \gamma. \tag{32}$$

The substitution of (31) into (25) leads to the following expression for the magnitude of the radiation pressure:

$$P(\tau) = -4\,\frac{\sigma_{\nu_0}}{c}\, x_1 \frac{dB}{d\tau}. \tag{33}$$

The magnitude of the radiation pressure can also be determined from Eqs. (25) and (28) for the case when the absorption coefficient depends on the thermal motion of atoms and the extinction of the radiation.

5. Three-Dimensional Medium

The results obtained above are easily extended to the case of a three-dimensional medium consisting of plane-parallel layers. We will assume that the velocity v is perpendicular to the layers and that $dv/d\tau$ = const and $dv/d\tau \geq 0$. In this case the intensity of the radiation moving at an angle $\cos^{-1}\mu$ to the normal is given by

$$I(\tau,\ x,\ \mu) = \int_0^{\tau} B(\tau')\,\alpha[x + \gamma\mu(\tau - \tau')] \exp\left\{-\int_0^{\tau-\tau'} \alpha(x + \gamma\mu z)\frac{dz}{\mu}\right\}\frac{d\tau'}{\mu}, \tag{34}$$

$$(0 < \mu \leqslant 1)$$

$$I(\tau,\ x,\ \mu) = -\int_{\tau}^{\tau_0} B(\tau')\,\alpha[x - \gamma\mu(\tau' - \tau)] \exp\left\{\int_0^{\tau'-\tau} \alpha(x - \gamma\mu z)\frac{dz}{\mu}\right\}\frac{d\tau'}{\mu}; \tag{35}$$

$$(-1 \leqslant \mu < 0).$$

The function $B(\tau)$ is given by

$$B(\tau) = \frac{A}{2}\int_{-\infty}^{+\infty} \alpha(x)\, dx \int_{-1}^{+1} I(\tau,\ x,\ \mu)\, d\mu + B_0(\tau). \tag{36}$$

Substituting (34) and (35) into (36), we obtain the integral Eq. (8), in which $K(t)$ is given by the following equation:

$$K(t) = A\int_0^1 \frac{d\mu}{\mu}\int_{-\infty}^{+\infty} \alpha(x + \gamma\mu t)\exp\left\{-\int_0^t \alpha(x + \gamma\mu z)\frac{dz}{\mu}\right\}\alpha(x)\, dx. \tag{37}$$

The approximate solution of this equation is Eq. (11), in which

$$\begin{aligned} L(\tau,\ \gamma) &= A\int_0^1 d\mu \int_{-\infty}^{+\infty} \alpha(x)\exp\left\{\int_0^{\tau} \alpha(x + \gamma\mu z)\frac{dz}{\mu}\right\} dx = \\ &= A\int_0^1 d\mu \int_{-\infty}^{+\infty} \alpha(x)\exp\left\{-\frac{1}{\gamma\mu^2}\int_x^{x+\gamma\mu\tau} \alpha(y)\, dy\right\} dx. \end{aligned} \tag{38}$$

In the case of a moving medium with a large velocity gradient (case I), we obtain from (38)

$$L(\infty,\ \gamma) = A\gamma\int_0^1 \left(1 - e^{-1/A\gamma\mu^2}\right)\mu^2\, d\mu. \tag{39}$$

For $A\gamma \ll 1$, this yields

$$L(\infty, \gamma) = \frac{1}{3} A\gamma. \tag{40}$$

In the case of a motionless medium (case II), from (38) we find

$$L(\tau, 0) = A\int_0^1 d\mu \int_{-\infty}^{+\infty} \alpha(x) \exp\left\{-\frac{\tau}{\mu}\alpha(x)\right\} dx. \tag{41}$$

If $\alpha(x)$ is given by Eq. (19), then

$$L(\tau, 0) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-x^2} \mathrm{Ei}_2(\tau e^{-x^2})\, dx. \tag{42}$$

The asymptotic expression for $L(\tau, 0)$ for large τ has the form

$$L(\tau, 0) = \frac{1}{2\tau\sqrt{\pi\ln\tau}}. \tag{43}$$

Let us calculate the pressure exerted by the L_α-radiation. Since the magnitude of the radiation pressure acting on a unit volume at a depth τ is equal to

$$P(\tau) = 2\pi\frac{\sigma_{\nu_0}}{c}\int_{-1}^{+1} \mu d\mu \int_{-\infty}^{+\infty} I(\tau, x, \mu)\, \alpha(x)\, dx, \tag{44}$$

the substitution of expressions (34) and (35) into (44) yields

$$\begin{aligned} P(\tau) = 2\pi\frac{\sigma_{\nu_0}}{c}\int_0^1 \mu d\mu \int_{-\infty}^{+\infty} \alpha(x)\, dx \; \times \\ \times \left\{\int_0^{\tau} B(\tau')\, \alpha[x + \gamma\mu(\tau - \tau')] \exp\left\{-\int_0^{\tau-\tau'} \alpha(x + \gamma\mu z)\frac{dz}{\mu}\right\}\frac{d\tau'}{\mu} - \right. \\ \left. - \int_{\tau}^{\tau_0} B(\tau')\, \alpha[x + \gamma\mu(\tau' - \tau)] \exp\left\{-\int_0^{\tau'-\tau} \alpha(x + \gamma\mu z)\frac{dz}{\mu}\right\}\frac{d\tau'}{\mu}\right\}. \end{aligned} \tag{45}$$

Let us investigate the case of a moving medium with a large velocity gradient. Taking for $B(\tau')$ the expression given by Eq. (24), as was done in § 4, we get

$$P(\tau) = -2\pi\frac{\sigma_{\nu_0}}{cA}\frac{dB}{d\tau}[\Phi(\tau, \gamma) + \Phi_{-}(\tau_0 - \tau, \gamma)], \tag{46}$$

where

$$\Phi(\tau, \gamma) = A\int_0^1 \mu d\mu \int_{-\infty}^{+\infty} \alpha(x)\, dx \int_0^{\tau} t\alpha(x + \gamma\mu t) \exp\left\{-\int_0^{t} a(x + \gamma\mu z)\frac{dz}{\mu}\right\}\frac{dt}{\mu}, \tag{47}$$

which, after some transformations, becomes

$$\Phi(\tau, \gamma) = A\int_0^1 \mu^2 d\mu \int_{-\infty}^{+\infty}\left\{1 - \left[1 + \frac{\tau}{\mu}\alpha(x)\right]\exp\left\{-\frac{1}{\gamma\mu^2}\int_x^{x+\gamma\mu\tau} \alpha(y)\, dy\right\} dx. \tag{48}$$

In the case when the absorption-coefficient contour is determined by the Doppler effect, the magnitude of the radiation pressure, as in § 4, is given by

$$P(\tau) \approx -\frac{8\pi}{3}\bar{x}_1 \frac{\sigma_{\nu_0}}{c}\frac{dB}{d\tau}, \tag{49}$$

where

$$\bar{x}_1 = 3\int_0^1 x_1(\mu)\,\mu^2\,d\mu, \tag{50}$$

and $x_1(\mu)$ is given by the relation

$$\int_{x_1(\mu)}^{\infty} e^{-x^2}\,dx = \gamma\mu^2. \tag{51}$$

6. Comparison of the Results with Those of Other Papers

In previous papers, as was already mentioned, particular cases of the L_α-radiation diffusion were investigated. Let us compare the results of these investigations with the results obtained above.

The author [3] has investigated the diffusion of L_α-radiation in a medium consisting of plane parallel layers for the case of a rectangular absorption-coefficient contour. It was also assumed that there is a large velocity gradient in the medium.

In the equations derived above, let us take the absorption coefficient to be rectangular. For convenience, let $A\gamma = \beta$. Since $A = \frac{1}{2}$ in the case of a rectangular contour, we have

$$\beta = \frac{1}{2u}\cdot\frac{dv}{d\tau}. \tag{52}$$

For Case I, according to Eq. (40), $L(\infty, \gamma) = \frac{1}{3}\beta$ if the medium consists of plane-parallel layers. From Eq. (11) we get

$$B(\tau) = \frac{3}{\beta}B_0(\tau). \tag{53}$$

From Eqs. (46) and (48) we find that the magnitude of the radiation pressure acting on a unit volume is

$$P(\tau) = -\frac{8\pi}{3}\frac{\sigma_{\nu_0}}{c}\frac{dB}{d\tau}. \tag{54}$$

Eqs. (52), (53) and (54) (with a change in notation) are the same as those obtained earlier [3] for the middle layers of the medium.

It is of interest to compare the results obtained on the assumption of a rectangular absorption-coefficient contour with the results obtained for other contours. As an example, let us take the Doppler contour, i.e., $\alpha(x) = e^{-x^2}$. In this case β is given by the equation

$$\beta = \frac{1}{\sqrt{\pi}\,u}\cdot\frac{dv}{d\tau}, \tag{55}$$

the quantity $B(\tau)$ is again given by Eq. (52), and the quantity $P(\tau)$ by Eq. (49). Even if Eqs. (52) and (55) are somewhat different from each other in appearance, in fact they are identical (since $d\tau = \sigma_{\nu_0}ds$, while the value of σ_{ν_0} for the Doppler contour and the value of σ_{ν_0} for a rectangular contour are in the ratio of $2 : \sqrt{\pi}$. Therefore, in both cases the values of the quantity $B(\tau)$ will also be the same. As regards the values of the quantity $P(\tau)$, from a comparison of (49) and (54) we can see that they differ from each other by the presence of the factor x_1. For γ of the order of 10^{-4}, this factor is approximately equal to three.

If we adopt any other form for the absorption-coefficient contour, then we will get the same value as before for the quantity β (this was shown in §3); therefore, the value of the quantity $B(\tau)$ will also remain unchanged. However, the magnitude of the radiation pressure can change considerably if the contour is changed.

A number of authors (see §1) have investigated the diffusion of L_α-radiation in a motionless medium for the case of a Doppler absorption-coefficient contour. An approximate solution of this problem follows from the substitution into Eq. (11) of the expression for the function $L(\tau,0)$ given by Eq. (20) in the case of a one-dimensional medium, or given by Eq. (42) in the case of a three-dimensional medium. A comparison of this approximate solution with the exact solutions obtained by Zanstra [4] and Koelbloed [7] shows that the approximate equations give results which are of the correct order of magnitude (the approximate results deviate from the exact results by less than a factor of 2-3). From physical arguments it follows that if a velocity gradient exists in the medium, the accuracy of Eq. (11) increases.

7. Application to Nebulae and Stellar Envelopes

The ionization of the hydrogen atoms from their ground state under the action of the stellar radiation and the subsequent recombinations result in the production of L_α-quanta in nebulae and stellar envelopes. The density of the ionizing radiation decreases with increasing distance from the star and, therefore, the function $B_0(\tau)$ also decreases. We can assume that

$$B_0(\tau) \sim \frac{1}{r^2} e^{-q\tau}, \tag{56}$$

where r is the distance of a given region from the star and $q\tau$ is the corresponding optical distance outside the Lyman-series limit (the magnitude of q is of the order of 10^{-4}). Since the solution which we have obtained above of the problem of L_α-quantum diffusion is based on the assumption that the function $B_0(\tau)$ is slowly varying inside the medium, we must take $q\tau < 1$. In other words, we should only consider those envelopes in which there is no HI region. If an HI region is present, it can happen that the L_α-radiation field in this region will be determined not so much by the production of L_α-quanta, but by the diffusion of L_α-quanta from an HII region into the region HI. It is obvious that the importance of this process will increase as the velocity gradient in the medium decreases. The problem of the diffusion of L_α-quanta in an evelope consisting of HI and HII regions must be investigated separately.

Let Δr denote the thickness of the envelope and r the distance of the envelope from the star. If Δr is small by comparison with r, then we can assume that the envelope consists of plane-parallel layers. The results obtained in § 5 are applicable in the case of such envelopes.

We have seen that the L_α-radiation field depends strongly on the quantity $\gamma\tau$. Let us put

$$\delta = \gamma\tau = \frac{\tau}{u} \cdot \frac{dv}{d\tau}. \tag{57}$$

When $\delta \gg 1$, the radiation field in the envelope approaches that for Case I; when $\delta \ll 1$, it approaches that for Case II. The degree to which this approximation holds depends on the form of the function $\alpha(x)$.

In order to evaluate the quantity δ, we note that $d\tau = \sigma_{\nu_0} dr$; we also take $\tau = \frac{1}{2}\sigma_{\nu_0}\Delta r$. We now have, instead of (57):

$$\delta = \frac{\Delta r}{2u} \cdot \frac{dv}{dr}. \tag{58}$$

As was shown earlier [3], in order to allow for the curvature of the envelope layers we should replace the quantity dv/dr by $dv/dr + 2v/r$. Since it is difficult to obtain an accurate estimate of dv/dr, we will assume that it is of the same order of magnitude as v/r. Therefore, the value of δ is

$$\delta = \frac{3}{2} \frac{\Delta r}{r} \frac{v}{u}. \tag{59}$$

Let us apply Eq. (59) to the case of planetary nebulae. Since the thickness of the nebula is only some tenths of its radius and the expansion velocity of the nebula is several times as large as the mean thermal velocity of the atoms, δ is of the order of unity. This means that the general form of the equations (and not their form for the limiting cases) should be used in the investigation of the L_α-radiation field in nebulae. In other words, it is necessary to take into account the escape from the medium of the quanta emitted in the line wings, as well as the escape of quanta due to the Doppler effect.

The situation is different in the case of nova envelopes. The expansion velocities in nova envelopes are considerably greater than the mean thermal velocities. Therefore, in this case $\delta \gg 1$. Thus, the L_α-radiation field in nova envelopes approaches the radiation field for Case I, i.e., it is determined primarily by the escape of quanta from the envelope as the result of the Doppler effect.

8. Concluding Remarks

I have recently become acquainted with a very interesting paper by Zanstra [11] which is devoted to the theory of emission lines in nebular spectra.

The problem of the diffusion of radiation in spectral lines in the case of a moving medium has been investigated in one of my papers [12]. In this it was assumed that the diffusion takes place with a frequency redistribution and a rectangular absorption-coefficient contour. The important theoretical quantity is the fraction of quanta which escape as the result of the Doppler effect, i.e., the quantity β. In the case of a rectangular contour, β is given by Eq. (52). Zanstra has calculated this quantity for the case of a Doppler contour and obtained Eq. (55). Since Eqs. (52) and (55) differ little from each other, Zanstra concluded that this confirms my results on the intensities and profiles of the emission lines. In fact, as we have seen, Eqs. (52) and (55) are not identical.

Zanstra has pointed out further, quite correctly, that for a moving medium the diffusion of radiation with frequency redistribution and a rectangular absorption-coefficient contour is equivalent to the diffusion of radiation without a change in the frequency. That is why, investigating further the diffusion of radiation with frequency redistribution in a motionless medium, I have taken the most general expression for the absorption coefficient which includes the attenuation of radiation and the thermal motion of the atoms. My results for a medium of infinitely large optical thickness (stellar atmosphere) were published in 1949 [13]. The results for a medium of finite optical thickness were not published then, because an excellent paper by Zanstra appeared at that time and was devoted to the same problem [4]. My results were published later as part of a more general investigation [10].

In his paper, Zanstra also raised the question of calculating the radiation pressure exerted by the L_α-radiation in a moving medium for the case of a Doppler contour of the absorption coefficient. This problem has been investigated above in §§4 and 5.

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ON THE NATURE OF THE OPTICAL EMISSION FROM THE CRAB NEBULA

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It is shown that the assumption that relativistic particles are still being ejected from the supernova of 1054 meets with insuperable difficulties. The formation of the well-known "wisp," which varies in brightness, in the central part of the Crab Nebula, as well as of the other variations of brightness of separate details of its "amorphous mass" are explained by fluctuations of the magnetic field. A comparatively small increase in the strength of the magnetic field will cause a considerable increase of the volume emissivity. The difficulty – the conservation of extremely energetic relativistic electrons during the "lifetime" of the nebula – is overcome.

The cause of the magnetic field fluctuations in the Crab Nebula is discussed and an interpretation of its morphological peculiarities given. The concentration and energy of relativistic electrons in the nebula have been recalculated on the basis of new data on its radio and optical spectra.

In 1953, we proposed a new interpretation of the optical emission with a continuous spectrum from the Crab Nebula [1]. According to this interpretation the optical emission from the Crab Nebula is determined by the radiation emitted by very energetic ($E \sim 10^{11}$-10^{12} ev) electrons moving in a magnetic field (called the "synchrotron mechanism"), while the less energetic electrons with an energy of $\sim 10^{8}$-10^{10}ev are responsible for the radio-frequency emission.

From this theory of the Crab Nebula, a completely new effect could be predicted: the polarization of the optical emission.

After this prediction was made [2], the polarization of light from the Crab Nebula was discovered in 1954 by Dombrovskii [3]. This new phenomenon was investigated further by a number of authors [4-8]. Of particular interest are the photographic observations of the polarization in the Crab Nebula carried out by Baade with the 200 inch telescope, as well as the recent photoelectric measurements of Hiltner [9]. These observations show that the light from the individual details of the so-called "amorphous mass" of the Crab Nebula is polarized to a large extent.

At the present time, the new theory of the emission from the Crab Nebula has been universally accepted and in some directions it has been developed further (see [7] and particularly [10]). In particular, S. B. Pikel'ner has explained, on the basis of this theory, the remarkable phenomenon of the secular acceleration of the filaments of the Crab Nebula, which was earlier considered to be completely unexplainable [10].

It is necessary to point out, however, that a series of problems associated with the emission and particularly with the nature of the Crab Nebula remain unsolved. At the same time, these problems are of exceptional interest in such important fields of physics and astrophysics, as the origin of cosmic rays, the nature of supernova explosions, and others.

In the present work an attempt is made to sort out some of these problems.

1. On the Nature of the Brightness Fluctuations in the "Amorphous Mass" of the Crab Nebula

First of all, we remark that the photographs of the so-called "amorphous mass" of the Crab Nebula, obtained with the 200 inch telescope [8], show on analysis that the term "amorphous mass" is unfortunate. In fact, for different points of the nebula we observe very large variations in the intensity of emission in the continuous spectrum. The so-called "amorphous mass" has a fairly well-developed structure, which is frequently in the form of fine filaments. We remark that the individual filaments of the "amorphous mass" in the majority of cases do not screen one another.

The fluctuations of the emissivity calculated per unit volume reach enormous values. Thus, for example, the bright changing formation observed by Baade in the central region of the nebula, called the "wisp" (see [7]), has a surface brightness at maximum 3.5 times as high as the mean brightness of the central region of the Crab Nebula, as can be evaluated from the data given in [7]. On the other hand, the maximum extension of the "wisp" along the line of sight does not exceed $2.5 \cdot 10^{16}$ cm. It follows directly from this that the volume emissivity in the "wisp" is approximately 300 times higher than the mean volume emissivity of the central region of the Crab Nebula, which value was used in [1], and later in [7], to calculate the concentration of relativistic electrons. On the other hand, the region at the center of the Crab Nebula surrounding the "wisp" has a surface brightness an order of magnitude lower than the average. Therefore, we must take into account the fact that in the central region of the Nebula the volume emissivity (in the continuous spectrum) varies by a factor of a hundred or even a thousand. Meanwhile, this important fact has not been taken into account up to the present in the "synchrotron" theory of the optical emission from the Crab Nebula.

The theory must also provide a reasonable explanation for the changeableness of the individual details in the "amorphous mass" of the Crab Nebula. The corresponding observational data, going back to Lampland [11], were summarized in [7]. Thus, for example, the "wisp" mentioned above forms fairly rapidly near the central star, exists for 2-3 months and then disappears, during which time it moves away from the central star with a velocity of ~25,000 km/sec. It is important to point out that such a fast movement of light-emitting matter in the Crab Nebula is not an exceptional phenomenon, a feature of the "wisp" only. The extended "boomerang-shaped" detail in the central region of the Nebula moved with a velocity of ~10,000 km/sec in the course of several years. Local variations of brightness in the northwest part of the Nebula have travelled with a velocity only slightly less than this. In general, we can say that velocities of the order of several thousand kilometers per second are a normal occurrence.

The first problem is to determine the reasons for the local variations of brightness of the Crab Nebula on the basis of the synchrotron-radiation theory. In doing this, naturally, we can start from two opposed assumptions:

a) The local increase of the brightness is due to an increase of the number of radiating relativistic electrons, while the magnetic field intensity is almost unchanged.

b) The local increase of the brightness is due to an increase of the magnetic field intensity.

The treatment of Oort and Walraven [7] is based on the first assumption, which seems to be attractive. They concentrated their attention to the analysis of the "wisp." They explained this phenomenon by a continuing emission of relativistic electrons from the former supernova. According to the ideas of Oort and Walraven, the necessity for the emission of relativistic electrons from the former supernova to continue up to the present time also follows from the comparatively short "lifetime" of the light-emitting electrons with $E \sim 10^{11}$-10^{12} ev in the magnetic field $H = 10^{-3}$ gauss, which was adopted in [7]. Such electrons must lose their energy in 180 years and, consequently, it is necessary to find some constant source to renew them. The idea that the emission of relativistic electrons from the former supernova takes place now was also held by S. B. Pikel'ner [10].

It appears to us, however, that this viewpoint encounters very serious objections. Firstly, it is hard to understand why the former supernova retains an enormous activity a thousand years after its explosion. Secondly, the serious limitations which have to be imposed at the same time on the nature of the magnetic field are hardly acceptable (see [10]). And, finally, the most important point is that this assumption encounters serious theoretical objections and is in contradiction with observational data. We will now consider this in greater detail.

1) From the analysis of the Crab Nebula photographs taken with different orientations of the polaroids

(see [7]) it follows that the magnetic field in the "wisp" is perpendicular to the direction of its motion. If we consider that the "wisp" is a cloud of relativistic particles emitted by the former supernova, then the magnetic field should be parallel to the direction of the "wisp." We can, of course, assume that there is a comparatively small (~10%) component of the field in a direction parallel to the motion of the "wisp." However, this assumption clearly appears to be artificial.

2) The "wisp" appears fairly quickly, at the outside in less than a month and, most likely, in less than two weeks (see [7]). On the other hand, the elongation of the "wisp" is ~12" or $2 \cdot 10^{17}$ cm. Taking into account that the magnetic field is parallel to the axis of the "wisp," we find that the time taken by a relativistic particle to traverse the "wisp" is $t = \frac{l}{c} = \frac{2 \cdot 10^{17}}{3 \cdot 10^{10}} = 6.6 \cdot 10^{6}$ sec or ~2.5 months. If we take into account that the particles move along a spiral in the direction of H, then t will be even longer, at least 4-5 months.

We can attempt to remove this difficulty, as well as the first, by assuming that the particles enter the "wisp" at right angles to its elongation, i.e., at right angles to the observed direction of the field. This again means that we must artificially assume that there is a small (~10%) component of the field in a direction from the star to the "wisp."

3) As was pointed out above, the "wisp" is far from being the only moving local increase of brightness observed in the Crab Nebula. Such individual formations, moving with a velocity of 5000-10,000 km/sec, are observed at considerably greater distances from the central star (~10^{18} cm) than the "wisp." Their appearance cannot be assumed to be the result of relativistic-electron emission from the former supernova. This follows even from the fact that the "wisp" only moves through $1\text{-}2 \cdot 10^{16}$ cm, after which it disappears. Of course such formations can be considered to be due to local "accumulations" of relativistic electrons, but this would mean that their nature is different from the nature of the "wisp." This, however, is hardly likely.

4) All of the above objections, although serious, are not decisive enough to disprove the assumption that the former supernova at the present time is still a powerful source of relativistic particles. As was indicated above, the objections can be overcome by the introduction of several artificial assumptions and only a detailed analysis of the original observational data can make these objections irrefutable. We will now present a decisive objection which, together with the above objections, will prove the inadequacy of the assumption that the former supernova continues to emit relativistic particles.

If we accept the viewpoint that the increase of the volume emissivity in the "wisp" (as also in the other similar formations) by a factor of several hundred is determined by an inflow of relativistic electrons into a certain region of space, while at the same time H remains constant, we encounter a fundamental difficulty. The fact is that, in developing the model of the Crab Nebula, we assumed that the energy density of the relativistic particles was equal to the magnetic energy density $(1/8\pi)H^2$ which is equal to the magnetic pressure. It is obvious that an increase of the concentration of relativistic particles in the "wisp," or in any other detail of the "amorphous mass," will lead to a rapid breakdown of the equilibrium. Consequently, the relativistic particles will not be retained by the magnetic field. No accumulation of relativistic particles of any importance can occur in a constant and comparatively weak field.

It appears that this difficulty can be eliminated if it is assumed that in the central region of the Crab Nebula the magnetic field intensity is considerably higher than that taken in [1] and [7] and that, on the average, the magnetic energy density is considerably higher than the energy density of the relativistic particles. Consequently, even when the density of relativistic particles increases by a factor of several hundred, the field can still retain them. However, if we accept this, then new and more serious difficulties arise. Firstly, the energy losses of the light-emitting electrons to radiation in the magnetic field rise sharply and this will make the problem of their "renewal" even more difficult. If this strong field is more or less constant within the central regions of the Crab Nebula, then it is completely incomprehensible why the relativistic electrons emitted by the star only begin to radiate in the region of the "wisp," at a distance of ~7" from the star. If we assume that the field is weak in the immediate vicinity of the "wisp," while in the region of the "wisp" it increases, then a new difficulty arises. As is known, for a relativistic particle moving in a magnetic field which is constant in time, the following relation is satisfied:

$$\sin^2\alpha = \text{const} \cdot H, \qquad (1)$$

where α is the angle between the direction of motion of the relativistic particle and the direction of the magnetic field. From (1) it follows that the relativistic particle cannot penetrate a region where H is sufficiently large. The assumption of the existence in the Crab Nebula of regions with a high value of the magnetic field intensity, by itself, is contrary to the viewpoint(a) and corresponds already to the opposite viewpoint (b).

Since the assumption (a) encounters very serious and apparently insurmountable difficulties, the opposite assumption(b) must be considered. In this case we must accept that in the Crab Nebula, for some reason or other, there are fluctuations in the magnetic field intensity which appear and disappear more or less rapidly. Let us see what will happen if the magnetic field intensity in some region of the Nebula increases by a factor m. Since the relativistic particles, moving in helical paths around the magnetic force lines, behave as if "tied" to the lines of force, an increase of the magnetic field ("concentration" of the lines of force) will lead to a corresponding increase of the concentration of relativistic electrons. If the differential energy spectrum of the relativistic electrons is represented by the usual expression,

$$N(E) = K/E^{\gamma}, \tag{2}$$

then the value of K will increase by a factor m when H increases by a factor m.

Further, according to the theory of synchrotron radiation, the emission from a unit volume, in a unit frequency interval, will be

$$\varepsilon_\nu \propto K H_{\perp}^{(\gamma+1)/2} \nu^{(1-\gamma)/2} \tag{3}$$

where $H_{\perp}$ is the magnetic field component perpendicular to the momentum of the relativistic electron. Consequently, if H is increased m times, the volume emissivity will increase by a factor $m^{(3+\gamma)/2}$.

There is also another factor which leads to an increase of ϵ_{ν} as H increases. As is known, for a relativistic particle in a magnetic field which changes with time and position, there exists an invariant of the motion:

$$P_{\perp}^{2}/H = \text{const}, \tag{4}$$

where $P_{\perp}$ is the momentum component perpendicular to the direction of the magnetic field. In the particular case when H is constant with time, this yields the Invariant (1). From (4) it follows that if the field increases in a certain region then, firstly, the angle α will increase so that $H_{\perp}$ will increase faster than H. Secondly, the energy of the particle will in general increase (because of the induction effect). The energy can increase by a factor as high as $(H_2/H_1)^{1/2}$. It is easy to show that this effect will lead to an increase of the volume emissivity by a factor of $m^{(\gamma-1)/2}$.

All of the three effects considered will lead to an increase of the volume emissivity by a factor n:

$$n = m^{\gamma+1} = \left(\frac{H_2}{H_1}\right)^{\gamma+1}. \tag{5}$$

For the optical region of the Crab Nebula spectrum, $\gamma \geq 4$ (see [12]). Consequently, if for example m = 3, then the volume emissivity will increase by a factor of more than 200. Therefore, the considerable local variations in the volume emissivity which are observed in the Crab Nebula are immediately explained by comparatively small (by a factor of 2-3) fluctuations of the magnetic field intensity. Consequently, there is no need to employ the assumption that the former supernova continues to emit relativistic particles to explain the increase of the volume emissivity. According to our point of view, the variation with time or position of the volume emissivity of the so-called "amorphous mass" of the Nebula is explained by the nature of the fluctuations of the magnetic field intensity. In particular, the same explanation must be used for the "wisp" in the central region of the Crab Nebula. In this case the fluctuation of the magnetic field intensity is a progressive wave of compressed force lines which exists for 2-3 months and then disappears. The "perpendicular" component of the magnetic field in the "wisp" can be estimated to be 10^{-3} oersted, while in the comparatively "dark" surrounding region $H_{\perp} \leq 3 \cdot 10^{-4}$ oersted. The time in which the relativistic electrons will lose their energy (more accurately the time in which their energy E will decrease to one-half its original value) is given by the expression:

$$t_{1/2} = \frac{0.0085}{H_{\perp}^{2} E}. \qquad (6)$$

If the magnetic field component perpendicular to the velocity is $H_{\perp} = 10^{-3}$ oersted, $t_{\frac{1}{2}}$ will be 190 years, which is several hundred times longer than the time during which the "wisp" exists. Consequently, we must conclude that the reason for the disappearance of the "wisp" is not the energy loss of the relativistic electrons, but the fading away of the magnetic field fluctuation. During this, the energy of each relativistic electron will decrease as the result of the Invariance Relation (4).

As the result of the considerations given above, we have to alter the current ideas on the energy loss by radiation of the relativistic electrons responsible for the optical emission from the Crab Nebula. According to these ideas, with $H_{\perp} = 5 \cdot 10^{-4}$ oersted (which according to [10] occurs in the central region of the Crab Nebula) the "light-emitting" electrons will lose one-half of their energy in 500 years. Actually, each relativistic electron as it wanders through the magnetic field of the Crab Nebula will be subject to extremely varied conditions. When it is in a region of magnetic field fluctuation it will suffer energy losses by radiation which considerably exceed the losses occurring in the absence of fluctuations. This eliminates the serious difficulty of having to explain the existence in the Crab Nebula of the very energetic "light-emitting" electrons which, because of the large energy losses, would not have survived for the last 900 years. According to the ideas developed above, the "light-emitting" electrons can be considered to have lost their identity. During the course of some 900 years, a particular relativistic electron need not emit optical radiation all of the time, i.e., it need not continuously have a sufficiently high energy. For example, let us consider the electrons with an energy $E > 4 \cdot 10^{10}$ ev; taking into account the spectrum of the "amorphous mass" of the Crab Nebula, we find that these electrons will be approximately 250 times more numerous than the electrons with $E > 2 \cdot 10^{11}$ ev [$N(E) \propto E^{-4.5}$]. The latter electrons produce the optical radiation. We can assume that most of the electrons with $E > 2 \cdot 10^{11}$ ev are in the magnetic field fluctuations. These fluctuations can exist for 2-3 months (as in the case of the "wisp") or, more frequently, several years (as in the northwestern region of the Nebula). After several years the electrons with $E > 2 \cdot 10^{11}$ ev will no longer be in the field fluctuations, their energy will decrease [see Eq. (4)], and they will no longer emit optical radiation. Therefore, there exists, so to say, an extensive "store" of relativistic electrons with $E > 4 \cdot 10^{10}$ ev which replenishes continuously the number of "light-emitting" electrons. It is clear that the time average of the radiation losses per electron for the electrons with $E > 4 \cdot 10^{10}$ ev will be 250 times smaller than that for the "light-emitting" electrons whose energy is higher than $\sim 10^{11}$ ev all of the time and which are moving in the appropriate field. This considerably reduces, if not eliminates, the difficulty associated with the "preservation" of relativistic electrons for a considerable length of time.

It is necessary, however, to point out that the question of whether the "light-emitting" electrons have been preserved from the time of the supernova explosion in 1054, as yet remains open. This question is associated with the analysis of the physical conditions which existed at the earliest stage of the development of the Crab Nebula when, for example, the magnetic field intensity in the Crab Nebula could have been considerably greater than it is now. Here we encounter the problem of the production of very high energy particles in the explosion of supernovae. Without attempting to consider this problem in the present work, we can put forward the hypothesis that the relativistic particles were not produced during the explosion, but at a later stage in the evolution of the nebula (see [11]).

2. On the Fluctuations of the Magnetic Field Intensity in the Crab Nebula

Let us now attempt to discuss the problem of the possible reasons for the observed fluctuations of the magnetic field intensity in the Crab Nebula. It is necessary to remark that this problem is very difficult and the considerations given below are only preliminary in nature.

First of all, let us note that the fluctuations of the magnetic field can be produced by the interaction of the magnetic field with some physical agency. Up to the present time it was considered that this agency can be the ionized gas in turbulent motion. The magnitude of the field was estimated from the well-known relation $\rho V^2/2 = H^2/8\pi$, where ρ is the density of the gas, V the velocity of the macroscopic motions which "trap" the lines of force of the magnetic field. V was usually taken to be ~ 300 km/sec, which corresponds to the observed turbulent velocity in the gaseous filaments of the Crab Nebula. It is obvious, however, that this value of V has been chosen arbitrarily, since the velocities of the turbulent motions in the filaments are clearly completely unrelated to the velocity of turbulent motions in the "amorphous mass" of the Crab Nebula.

An important question arises in this connection: what is the nature of the gas in the inner ("amorphous") region of the Crab Nebula? Cannot we assume that this gas is nothing else but an association of relativistic particles?

Of great importance for the problem investigated are the recent observations at low frequencies of the radio-frequency emission from the Crab Nebula [13]. According to [13], at a frequency of 26.75 Mc the spectral density of the flux of radioemission from the Crab Nebula is $F_\nu = 8.5 \cdot 10^{-23}$ w/m^2·cps. On the other hand, according to very reliable measurements, $F_\nu = 1.85 \cdot 10^{-23}$ w/m^2·cps at a frequency of 81.5 Mc. From this follows that in the frequency interval $81.5 > \nu > 26.75$ Mc, $F_\nu \propto \nu^{-1.3}$. This means that the exponent in the differential energy spectrum of the comparatively soft electrons is $\gamma = 3.6$. It is quite probable that for softer electrons γ is even larger.

We assume that the average value of the magnetic field intensity in the Crab Nebula is $H = 3 \cdot 10^{-4}$ oersted [10], its angular dimensions are $\Omega = 1.5 \cdot 10^{-6}$, and that the intensity of radioemission at a frequency of $\nu = 26.7$ Mc is $I_\nu = 5.5 \cdot 10^{-14}$ cgs units; then from the basic equation of the synchrotron radiation theory

$$I_\nu = \frac{3}{\pi}(2\pi)^{(1-\gamma)/2}\frac{e^3H}{mc^2}u(\gamma)RK\nu^{(1-\gamma)/2}; \qquad (7)$$

with $\gamma = 3.6$, $R = 2 \cdot 10^{18}$ cm, $u(\gamma) = 0.8$, we find that $K = 4 \cdot 10^{-16}$ cgs units. The average concentration of relativistic electrons with an energy $E > E_0$ will be equal to

$$N(E > E_0) = \int\limits_{E_0}^{\infty} N(E)\,dE = K\int\limits_{E_0}^{\infty}\frac{dE}{E^{3.6}} = \frac{K}{2.6}\cdot E_0^{-2.6}, \qquad (8)$$

which gives the result that $N(E > 10^7 \text{ ev}) \approx 8 \cdot 10^{-4}$ cm^{-3} and $N(E > 3 \cdot 10^6 \text{ ev}) \approx 1.3 \cdot 10^{-3}$ cm^{-3}. The question of to what energies we can extrapolate the energy spectrum of the relativistic electrons, of course, remains open. But at any rate it is clear that the average concentration of soft relativistic electrons, particularly in the regions of the magnetic field fluctuations, can be of the order of 10^{-2} cm^{-3}. Their pressure will be of the order of 10^{-8} erg·cm^{-3}, i.e., they will still be retained by the magnetic field.* Such a high concentration of soft relativistic particles is already not very different from the concentration of the interstellar gas in the space between gas clouds. Therefore, we can assume, in general, that gas in the usual sense, if it is present at all in the "amorphous mass" of the Crab Nebula, has no influence on the nature of the magnetic field.

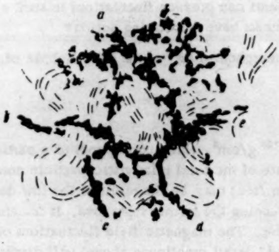

Fig. 1

However, it is improbable that the magnetic field intensity and its fluctuations inside the Crab Nebula are governed by the motion of the gas consisting of relativistic particles. If this were so, then it is hard to imagine how the relativistic particles have been retained in the Crab Nebula for as long as 900 years. It would also be difficult to understand the remarkable stability of the general appearance of the Crab Nebula, in spite of the considerable fluctuations of the brightness of separate regions. Apparently, in the inner part of the Crab Nebula the influence of the magnetic field is more important than that of the moving gas. The high degree of uniformity of the field in the inner part of the Nebula also speaks in favor of this assertion. On the other hand, on the periphery of the Nebula the degree of polarization of the optical radiation is not high [7], which indicates that the magnetic force lines are very tangled. It is natural to associate this occurrence with the filaments of gas which, as is known, are on the periphery of the nebula. Consequently, the magnetic field of the "amorphous mass" must be tightly bound to the system of expanding gas filaments.

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* The concentration and energy of the most energetic "light-emitting" electrons must be re-estimated, because according to [12] in the optical and near-infrared regions of the spectrum, $5 > \gamma > 4$. Taking $\gamma = 4.5$, $H_\perp = 3 \cdot 10^{-4}$, we find that the concentration of "light-emitting" electrons is 2.5 times smaller than the concentration with $\gamma = 3$, while the energy density is 6.4 times smaller. The total energy of "light-emitting" electrons with $E > 3 \cdot 10^{11}$ ev in the Crab Nebula is approximately $2 \cdot 10^{46}$ ergs, i.e., small.

A good illustration of what has been said above can be obtained by considering the distribution of gas filaments and magnetic force lines on the periphery of the Crab Nebula (see Fig. 1, where the direction of the magnetic force lines was determined from polarization measurements). The following observation is very striking: in the southeastern region of the Nebula the magnetic force lines clearly "avoid" the filaments and appear to "flow" around them. The bending of the force lines in the spaces between the filaments into gigantic arcs is very marked. This characteristic behavior can be seen everywhere on the periphery of the nebula. It appears that in this case we observe the tendency of the magnetic field and the relativistic particles moving in this field to "break through" in those regions of the nebula where this can be most easily done, i.e., in the space between the filaments. The fact that at the ends of the arcs the field is tangled up by the gas filaments prevents the field from "breaking through." The only result is that the field becomes highly deformed and tends to drag the filaments with it. But the energy of the field is not sufficient to disperse the system of gas filaments, although it does produce some acceleration of individual regions in the filaments. S. B. Pikel'ner, on the basis of this phenomenon, was able to estimate the total mass of the Crab Nebula [10]. It appears that the high stability of the dark "bay" in the southeastern part of the nebula can be explained by the presence of the thick filaments a, b and c which determine the character of the field.

Thus, the expanding system of gas filaments drags the magnetic field with it, at the same time trapping the magnetic force lines and continually deforming them. Under these conditions hydromagnetic waves can propagate along the system of magnetic force lines. As the result of reflections from the periphery of the nebula, shock waves can be formed, at the nodes of which the volume emissivity will have its maximum value. These waves must disperse more or less rapidly.

If we accept this point of view, then we see that the origin of the magnetic field fluctuations in the Crab Nebula must be due to the system of expanding gas filaments. However, it must be pointed out that it is very difficult to explain the very small fluctuations of the field observed in the "wisps." For it is not easy to imagine how the large-scale interactions of comparatively distant gas filaments can produce fluctuations in such a small volume. Is it possible that in this case relatively close, small filaments have a focussing action?

The very rapid velocity of the brightness variations in the "amorphous mass" of the Crab Nebula can be interpreted as the velocity of hydromagnetic waves:

$$V = H\sqrt{4\pi\rho}.$$

Taking $V \sim 10^9$ cm/sec, $H = 3 \cdot 10^{-4}$ oersted, we find that $\rho \sim 10^{-26}$ g/cm^3, which corresponds to a particle concentration of $\sim 10^{-2}$ cm^{-3}. This is in agreement with the estimate of the total relativistic-particle concentration made above. The high velocity of the "wisp" ($\sim 2.6 \cdot 10^9$ cm/sec) may be the result of the low density of the relativistic gas in the very central region of the nebula surrounding the former supernova. It is natural to associate the short lifetime of the "wisp" with its small dimensions. The magnetic field fluctuations occurring in bigger regions of space (for example, the "boomerang-shaped" detail mentioned above) will disperse more slowly.

We have come to the conclusion, then, that the magnetic field in individual regions of the Crab Nebula will rapidly change its character from time to time. All the time it will have a tendency to "break through" the network of the gas filaments. If, for some reason or other, conditions favorable for such a "breakthrough" are established, the field in this "weak" spot will be rapidly deformed. The system of magnetic force lines in the Crab Nebula is sufficiently elastic, which guarantees a high velocity for the observed variations.

We have only analyzed some of the properties of the magnetic field in the Crab Nebula. In doing this we did not investigate the difficult problem of the origins of this field. However, it must be pointed out that once the properties of the magnetic field in the present-day Crab Nebula are sufficiently understood, the problem of the origin of this field can be put on a scientific basis.

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ON THE RADIO EMISSION OF CORONAL CONDENSATIONS

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The position of the center of solar radio emission on 10 and 50 cm is compared with coronal disturbances in the green line 5303 A. Results of radio observations obtained during 1952-1953 at the Crimean scientific station of the FIAN are used.

It is established that coronal disturbances in the green line are responsible for the appearance of additional sources of radio emission on the sun.

1. Introduction

It is now definitely established that solar radio emission in the decimeter and meter ranges originates in the solar corona and hence by studying this emission one may obtain new information about the processes which take place in the outer regions of the sun.

Solar radio emission in the decimeter range (with which we shall be concerned below) may be looked upon as consisting of the following components.

1. Radio emission of a quiet sun. This is due to the thermal emission of the corona.
2. A component which varies slowly with time. This component is associated with long-lived solar formations such as spots, coronal disturbances, etc.
3. Bursts of radio emission associated with fast solar processes such as, for example, chromospheric flares.

Some results of an investigation of the second of the above components are reported in the present paper.

The appearance and disappearance of the slowly varying component manifests itself through a corresponding increase, and a subsequent decrease, in the intensity of radio emission from separate parts of the solar corona. The slowly varying component does not change very much during a single day and hence to study it it is sufficient to carry out only one or two observations per day. In order to study this component, it is usual to measure the intensity of radio emission of the sun. A comparison of the intensity of radio emission with some optically observed solar formation gives a basis for deducing the corresponding correlation. Observations during eclipses also give useful information in this connection. Before we give an account of our own results, let us briefly consider the results of other workers.

It has long been noted during observations of solar radio emission that the intensity of this emission is not distributed uniformly over the solar disc. There are on the solar disc separate active formations which are the seat of increased radio emission. In particular, this fact was mentioned by Khaikin and Chikhachev [1], who carried out observations of the solar radio emission during the 1947 eclipse. Vitkevich and Chikhachev have also established the nonuniform distribution of radio brightness over the solar disc [2]. This was also pointed out by Troitskii, Rakhlin, Bobrik, and Zelinskaia [3]. Foreign authors have also noted, during observations of solar emission carried out during eclipses, that the fall in brightness is not smooth but includes considerable fluctuations which may be explained by the presence of additional sources of radio emission.

Attempts have been made, in a number of papers, to compare these sources of radio emission with active solar formations observed optically. Early papers of this type associated local sources of radio emission with sunspots. Thus, for example, Covington [4], using considerable observational material, concluded that there exists a close connection between the intensity of solar radio emission on 10.7 cm and the sunspot area (correlation coefficient >0.8). The same conclusion was reached by Reber [5] in connection with observations on 62.2 cm. The material obtained by Covington on 10.7 cm was re-examined by Waldmeir [6], who found that the variable component of solar radio emission is due to regions of increased density of coronal matter above large groups of spots, i.e., the so-called coronal condensations.

On the other hand, it was pointed out that there are reasons to expect a connection between increased radio emission and increased intensity of separate monochromatic coronal lines. Shklovskii [7] pointed out that it would be useful to carry out simultaneous observations of the solar radio emission in the decimeter range and of the corona itself in monochromatic light.

O'Brien [8], who carried out observations on 1.4 m, found a connection between time variation of the dimensions of a source of radio emission and the time behavior of the mean intensity of the green coronal line 5303 A. Coutrez, Koeckelenbergh, and Pourbaix [9], who carried out observations during the 1952 solar eclipse, have noted a connection between the localization of sources of solar radio emission and regions of maximum intensity of the coronal line 5303 A when they were working on a frequency of 169 Mc/s. Zaffineur, Michard, Pecker and Vauquis [10] have shown that the distribution of radio brightness on 55 cm may best be characterized by isophots of the green corona.

Beginning at the end of 1952, a series of observations of solar radio emission on 10 and 50 cm was carried out at the Crimean scientific station of FIAN. In view of the fact that an accurate investigation of the intensity of the emission is difficult for well-known technical reasons, our observations were based on a somewhat different method, namely, on the determination of the position of the effective center of solar radio emission.

The very first observations showed that the position of the center of solar radio emission on these wavelengths is not constant but changes from day to day by a measurable amount. Quite naturally, these results could have been explained by the appearance and motion along the solar disc of additional sources of radio emission. When compared with the motion of sunspots these results gave a definite correspondence, so that it was possible to conclude that the appearance of these additional sources of radio emission is connected with the spots [11]. However, a number of characteristic properties indicate that there was a difference between the overall picture of the appearance and disappearance of spots at the solar limb and the radio data. In this connection an attempt was made to compare the above radio observations with the data on the intensity of the green line of the corona. A brief report on this work was published earlier [12] and a fuller account is given in the present paper.

2. Method of Observation and the Treatment of Results

Observations on the wavelength of 50 cm were carried out at Alusht using a double antenna interferometer. A differential modulation radiometer served as the receiving device. The base of the interferometer was placed in the east-west direction. The dimensions of the base were $D = 35.95$ m $= 71.3\ \lambda$; the angular aperture of the central lobe was $\Delta\varphi = 47'$. The relation between the dimensions of the base and the source was such that a clear interference pattern was obtained for the sun. Observations were made for three hours daily, at a time close to the culmination of the sun.

Observations on 10 cm were carried out on the "Koshka" mountain near Simeiz using a single mirror-reflector. The reflector was in the form of a paraboloid of revolution having a diameter of 7.5 m; the angular width of the main lobe was $\Delta\varphi = 45.8'$. The antenna was directed towards the south and readings were taken during the few minutes while the sun was passing through the central meridian. Fig. 1 shows a typical record of solar radio emission on 10 and 50 cm. The above apparatus was more fully described in [13].

In the treatment of the observations obtained on 50 cm we first determined for each day the time of culmination of the effective center of solar radio emission, i.e., the time at which the solar center passed through the central lobe of the antenna radiation pattern. For this purpose the time corresponding to the maximum of the central lobe was measured directly from oscillograms. Let us denote this time by t_1 (for greater accuracy, this time was found by averaging over the times found for lobes which were symmetrical with respect to the central lobe). Then using astronomical tables, the culmination time to the center of the visible solar disc was calculated, and hence the difference $\Delta t = t_1 - t_0$, i.e., the time difference between the culminations of the observed center and the visible center of the solar disc, was found. If $\Delta\vartheta$ denotes the angular shift between the center of the sun and the center of the visible disc projected onto the direction of the base, then it is easy to show that

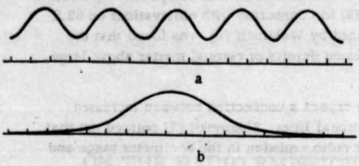

Fig. 1. Curves of recorded solar radio emission on the following wavelengths: a) λ = 50 cm (interferometer observations); b) λ = 10 cm (record obtained with a single and very directional antenna with D = 7.5 m). Minutes are shown along the axes.

in the case of a horizontal base placed along the east-west direction we have

$$\Delta\beta = (A_1 - A_0)\sin z,$$

where $\underline{z}$ is the zenith distance, A_1 and A_0 are the azimuths of the sun corresponding to t_1 and t_2. A similar treatment of the oscillograms gives the value of $\Delta\beta_{50}$ for each day of observations. These values are represented by the continuous line in Fig. 2a.

In the treatment of the observations on 10 cm it was necessary, as in the previous case, to determine from the oscillograms the instant of time at which maximum intensity occurs, which corresponds to the instant when the center of the sun passes through the axis of the reflector. As before, let this instant be t_1, so that having found t_0 and $\Delta t = t_1 - t_0$, we can determine the displacement of the effective center of the sun in the east-west direction.

As a result of such a treatment we obtain for each day the values of $\Delta\beta_{10}$, and these are represented by the continuous line in Fig. 2b.

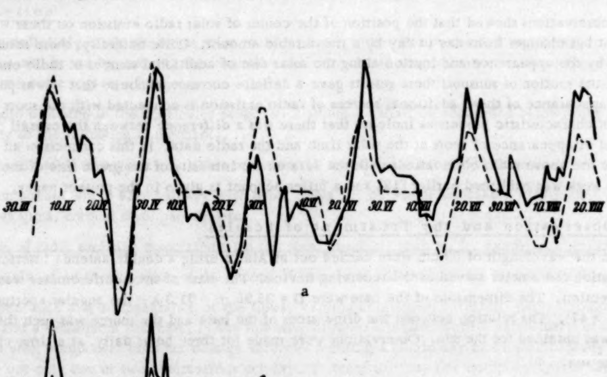

Fig. 2. $\Delta\beta_{50}$, $\Delta\beta_{10}$ (continuous line) – displacement of the effective center of solar radio emission on 10 and 50 cm, respectively; M_{20}, M_{25} (dotted line) – moments relative to the vertical which passes through the center of the solar disc obtained for regions of the green corona bounded by the isophots 20 and 25.

The oscillations of the position of the effective center of radio emission of the sun which take place about the position of the geometrical center of the visible solar disc and which were determined by measuring $\Delta\beta$, were compared with the motion over the disc (due to the rotation of the sun) of bright regions in the green corona on 5303 A. We used data on the emission of the green corona which were given on the synoptic charts in [14], while in the detailed treatment we used the period when only one coronal region was present on the solar disc. This period was April 1, 1953 to August 31, 1953.

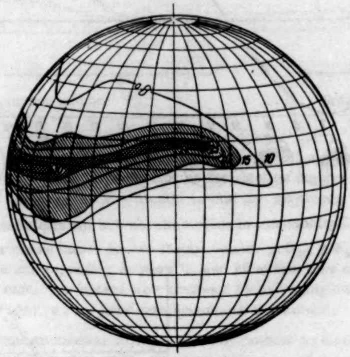

Fig. 3. Solar chart for August 1953, showing isophots of the green coronal line.

In accordance with these data, charts were constructed for each day showing the position and the dimensions of the coronal regions (Fig. 3).

The quantity $M = s \times l$ was taken as the quantitative characteristic of the position and the dimensions of these regions, where $\underline{s}$ is the area on the disc bounded on the given day by one of the isophots of the green corona and is expressed in cm^2, and l is the distance between the center of the corresponding region and the geometrical center projected onto the direction of the base and is expressed in solar radii. Since the base was in the east-west direction, it follows that the plane normal to the base is vertical and the moment of coronal regions was calculated relative to the vertical line passing through the center of the sun. The moments were calculated for each of the isophots 10, 15, 20, 25, 30 and 35 separately. The values of the moments of each of the coronal regions bounded by the isophots 20 and 25 are shown in Figs. 2a and 2b.

3. Discussion of Results

As can be seen from Figs. 2a and 2b, there is a close connection between changes in the position of the effective center of solar radio emission on both 10 and 50 cm and the moments of coronal regions in the line 5303 A. It is clear from the curves representing the moments of coronal formations that the observational period which was used was characterized by the presence of one coronal region of considerable dimensions which gradually weakened from turn to turn. The periodicity associated with the rotation of the sun is also noticeable in the displacement of the center of radio emission.

Fig. 4 shows, for a single turn, the dependence of the displacement of a coronal region, as the sun rotates about its axis, upon the change in the position of the center of radio emission. As soon as the active region appears from behind the limb the center of radio emission moves to the left (down, on the graph) and as the region moves towards the center of the sun the displacement is less, and then reappears again with a different sign.

In order to obtain more detailed information from these results, correlation coefficients were calculated

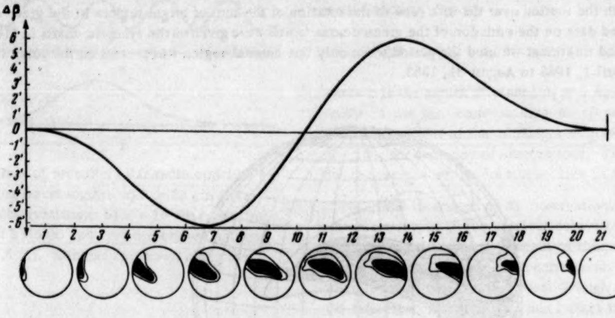

Fig. 4. The connection between the displacement of radio emission and the displacement of active coronal region over the solar disc.

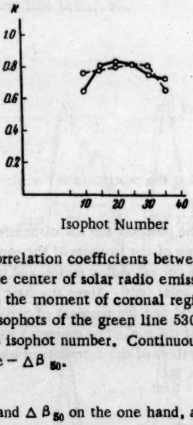

Fig. 5. Correlation coefficients between the displacement of the center of solar radio emission on 50 and 10 cm and the moment of coronal regions bounded by different isophots of the green line 5303 A as a function of the isophot number. Continuous line – $\Delta\beta_{10}$; dotted line – $\Delta\beta_{50}$.

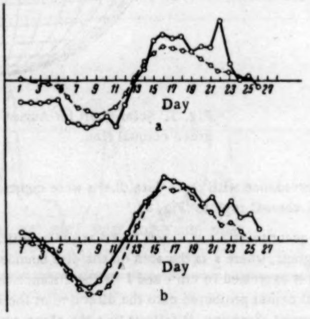

Fig. 6. The dependence of moments of coronal regions and the displacement of the center of solar radio emission upon time within the 27-day period of rotation of the sun about its axis. Averaged over a few periods. a) Continuous line – $\Delta\beta_{10}$; dotted line – M_{25}; b) continuous line – $\Delta\beta_{50}$; dotted line – M_{20}.

for $\Delta\beta_{10}$ and $\Delta\beta_{50}$ on the one hand, and the moment M of coronal regions bounded by different isophots on the other. Results of these calculations are shown in Fig. 5. As can be seen from this figure, for the wavelength of 50 cm the correlation coefficient is 0.65 for the isophot 10, it reaches its maximum value of 0.83 in the case of the isophot 20, and then for the isophots 30 and 35 it becomes less again. It is therefore clear that the region bounded by the isophot 20 corresponds to the highest correlation coefficient and may be taken as the effective region of radio emission on this wavelength.

On 10 cm wavelength the maximum correlation coefficient corresponds to the isophots 25-30 and is equal to 0.82. We note that, using analogous calculations for the same period, the correlation between the displacement of the center of radio emission and the moment of sunspots turns out to be considerably smaller and equal to 0.52 for 10 cm wavelength and 0.38 for 50 cm wavelength. A comparison of the curves obtained for 10 and 50 cm shows that when the wavelength is 50 cm the dimensions of the radiating region are somewhat larger than in the 10 cm case. This may be explained by some sort of "saturation" of radio emission on 50 cm (optical

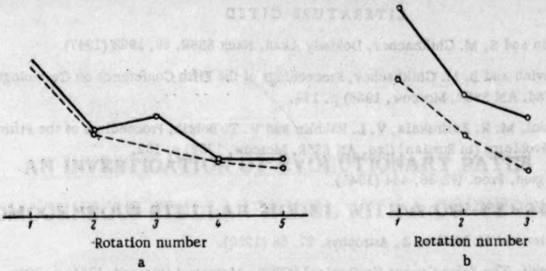

Fig. 7. The dependence of the maximum displacement of the center of solar radio emission and maximum changes in the moment of coronal regions for a rotation on the rotation number: a) λ = 50 cm; isophot 20; continuous line – $\Delta\beta_{50}$; dotted line M_{20}; b) λ = 10 cm; isophot 25; continuous line – $\Delta\beta_{10}$; dotted line M_{25}.

thickness ≥ 1). Fig. 6 gives the mean curves for the displacements $\Delta\beta_{10}$, $\Delta\beta_{50}$ and the moments of coronal regions M_{20}, M_{25} bounded by the corresponding isophots 20 and 25 as functions of time within the 27-day period of rotation of the sun about its axis. The curves were obtained by averaging over all the complete rotations which were studied. As can be seen, a very good correspondence is obtained.

Fig. 7 gives the data on the attenuation in the activity of coronal formations as a function of the rotation number. As can be seen from this figure, the reduction in the intensity of the green line is closely connected with the decrease in radio emission. These results show that there is a very close correlation between the intensity of the green coronal line and the intensity of radio emission in the decimeter range.

Thus radio observations of the sun give information on coronal activity in the 5303 A line without resort to optical observations.

The quantities $\Delta\beta_{10}$ and $\Delta\beta_{50}$ are also fairly well correlated (correlation coefficient = 0.68). Using the data obtained above, it is possible to obtain an average value for the coefficient of proportionality between $\Delta\beta_{10}$ and $\Delta\beta_{50}$. Calculations yield the following result:

$$\overline{\Delta\beta}_{10} = 1.15\overline{\Delta\beta}_{50}. \qquad (*)$$

We look upon the displacement of the center of solar radio emission as a result of the appearance of additional sources on the solar disc. If the intensity of these additional sources is characterized by the quantity $\Delta T\,\Omega$ where ΔT is their temperature and Ω is the solid angle, then the displacement of the center of gravity $\Delta\beta$ is proportional to $\Delta r \Delta T\,\Omega / T_{\odot}$, where $T_{\odot}$ is the effective temperature of the sun and Δr is the distance. Thus it follows from expression (*) that, on the average, the following expression holds for the radio emission of coronal condensations on 10 and 50 cm:

$$\left(\frac{\Delta r \Delta T \Omega}{T_{\odot}}\right)_{10} = \left(\frac{\Delta r \Delta T \Omega}{T_{\odot}}\right)_{50} \cdot 1.15,$$

hence, assuming that $\Delta r_{10} \approx \Delta r_{50}$ and $\Omega_{10} \approx \Omega_{50}$, we find on the average that $T_{\odot\,10} / T_{\odot\,50} = 0.15$, and thus

$$\Delta T_{10} \approx 0.17\,\Delta T_{50}.$$

This relation characterizes the average relationship between the effective temperatures of coronal regions on the above wavelengths.

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AN INVESTIGATION OF EVOLUTIONARY PATHS FOR A HOMOGENEOUS STELLAR MODEL WITH A CONVECTIVE CORE

G. Ruben and A. Masevich

In this paper we investigate the possibility of applying the homogeneous model of a gaseous star with a convective core in order to explain the different branches of the Hertzsprung-Russell diagram. The evolutionary paths for a homogeneous star are calculated for the case of variable mass (with the assumption of various possible mass-variation laws) and for the case of constant mass. The effect of the individual factors (the relative abundance of the heavy elements Z, the parameter ϵ_0 appearing in the energy generation law, etc.) on the form of the evolutionary paths is analyzed.

A detailed investigation of the model of a star with a convective core and the absorption law $k = k_0\rho^{0.875}T^{-3.5}$, as well as the calculation of the possible evolutionary paths for such a model in the case of a homogeneous and a nonhomogeneous chemical composition, was carried out earlier by one of the authors [1, 2] and applied to the evolution of main-sequence stars. In the present paper we investigate the possibility of explaining the structure and evolution of stars belonging to the other branches of the Hertzsprung-Russell diagram in terms of such a model.

We discuss the model with complete mixing, i.e., with the same chemical composition in the envelope as in the convective core. The carbon-cycle reaction is taken to be the source of energy:

$$\varepsilon = \varepsilon_0 XZ\rho T^n. \tag{1}$$

The continuous evolution (as the result of a gradual conversion of hydrogen into helium) of such a model is investigated in the case of a constant and a variable mass.

1. The Evolutionary Paths in the Case of the Homogeneous Model with Variable Mass

Let the star of mass $\mathfrak{M}_0$ have a luminosity L_0 and radius R_0 when $\mu = \mu_0$.

Then, using the following notation:

$$\mathfrak{M}_1 = \frac{\mathfrak{M}}{\mathfrak{M}_0};\; L_1 = \frac{L}{L_0};\;\; R_1 = \frac{R}{R_0}, \tag{2}$$

and taking n = 17, we get in the case of our model (see [1])

$$\frac{\mathfrak{M}_1^{43/8}}{L_1 R_1^{7/8}} = \left(\frac{\mu_0}{\mu}\right)^{7.5} \frac{1+X}{1+X_0}; \tag{3}$$

$$\frac{\mathfrak{M}_1^{19}}{L_1 R_1^{20}} = \left(\frac{\mu_0}{\mu}\right)^{17} \frac{X_0}{X}. \tag{4}$$

In addition to Eqs. (3) and (4), we have a further equation which characterizes the variation of the mass:

$$L_1 = \mathfrak{M}_1^{\gamma}. \tag{5}$$

In [2] γ was taken equal to the exponent of the mass in the mass-luminosity relationship for the main sequence stars; in our case, γ, in general, can have any value. The mean molecular weight is given by the equation:

$$\frac{\mu}{\mu_0} = \frac{X_0 + 0.6 - 0.2\,Z_0}{X + 0.6 - 0.2\,Z}, \tag{6}$$

where X is the relative abundance of hydrogen by weight and Z is the relative abundance of all the elements heavier than helium.

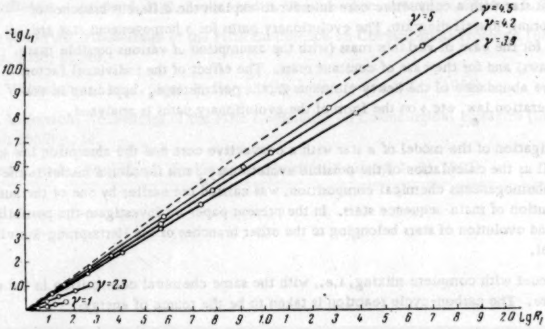

Fig. 1. Log L_1 as a function of log R_1 for various values of γ (Z = 0.01).

Eqs. (3), (4), (5), and (6) yield

$$\lg \mathfrak{M}_1 = (153\,\gamma - 767)^{-1} \lg \mathfrak{M}_1^{\bullet}; \tag{7}$$

where

$$\lg \mathfrak{M}_1^{\bullet} = 1081 \lg \frac{\mu}{\mu_0} + 160 \lg \frac{1 + X_0}{1 + X} + 7 \lg \frac{X_0}{X}; \tag{8}$$

$$\lg R_1 = \frac{1}{153}\,(107 \lg \mathfrak{M}_1 + 8 \lg R_1^{\bullet}), \tag{9}$$

in which

$$\lg R_1^{\bullet} = \frac{19}{2} - \lg \frac{\mu}{\mu_0} - \lg \frac{X_0}{X} - \lg \frac{1 + X_0}{1 + X}. \tag{10}$$

The calculations have been carried out for the following values of the parameters: γ = 1, 2.3, 3.9, 4.2, 4.5 and Z = 0.01, 0.1, 0.2, 0.4.

TABLE 1

	γ	1.0			2.3			3.9			4.2			4.5		
	X	$-\lg \mathfrak{M}_1$	$-\lg R_1$	$-\lg L_1$	$-\lg \mathfrak{M}_1$	$-\lg R_1$	$-\lg L_1$	$-\lg \mathfrak{M}_1$	$-\lg R_1$	$-\lg L_1$	$-\lg \mathfrak{M}_1$	$-\lg R_1$	$-\lg L_1$	$-\lg \mathfrak{M}_1$	$-\lg R_1$	$-\lg L_1$
$Z = 0.01$	0.9	0.05092	0.02607	0.05092	0.07533	0.04314	0.17326	0.18174	0.11794	0.70879	0.25133	0.1663	1.05539	0.39832	0.2691	1.79244
	0.8	0.10958	0.05633	0.10958	0.16209	0.09306	0.37281	0.39638	0.25686	1.54588	0.54085	0.3579	2.27157	0.85714	0.5791	3.85713
	0.7	0.17427	0.08924	0.17427	0.25630	0.14731	0.58949	0.61674	0.40628	2.44429	0.85521	0.5622	3.59188	1.35534	0.9150	6.09903
	0.6	0.24181	0.12501	0.24181	0.35769	0.20605	0.82269	0.87524	0.56765	3.41344	1.19349	0.7906	3.01266	1.89145	1.2767	8.51152
	0.5	0.31662	0.16400	0.31662	0.46497	0.26775	1.06943	1.14517	0.74322	4.46616	1.56270	1.0355	6.56334	2.47658	1.6746	11.14461
	0.4	—	—	—	—	—	—	2.13253	0.39077	8.31687	2.92012	1.94159	12.27			
$Z = 0.1$	0.9	—	—	—	—	—	—							—	—	—
	0.8	0.06022	0.03095	0.06022	0.17094	0.11915	0.39213							0.47012	0.31824	2.11959
	0.7	0.12470	0.06415	0.12470	0.35224	0.24598	0.81015							0.97539	0.65908	4.38925
	0.6	0.19464	0.10036	0.19464	0.54637	0.38182	1.25780							1.52247	1.02897	6.85112
	0.5	0.27057	0.13996	0.27507	0.75642	0.52806	1.73977							2.11639	1.43083	9.52375
	0.4	—	—	—	1.60480	1.05274	3.46104							—	—	—
$Z = 0.2$	0.9	—	—	—	—	—	—							—	—	—
	0.8	—	—	—	—	—	—							—	—	—
	0.7	0.06547	0.03366	0.06547	0.09684	0.05559	0.22273							0.51211	0.34601	2.30445
	0.6	0.10948	0.05145	0.10948	0.20172	0.11596	0.46396							1.06670	0.72088	4.80015
	0.5	0.16912	0.07948	0.16912	0.31586	0.18211	0.72648							0.67030	1.12933	7.51635
	0.4	—	—	—	0.44071	0.25460	1.01375							—	—	—
$Z = 0.4$	0.5	0.07712	0.04009	0.07712										0.60323	0.40803	2.71454
	0.4	0.16163	0.08446	0.16163										1.26426	0.85558	5.68917
	0.3	0.25572	0.13426	0.25572										2.00021	1.35476	9.00094

TABLE 2

γ	1.0			2.3			3.9			4.2			4.5		
Z	$\lg \mathfrak{M}_0$	$\lg R_\odot$	$\lg L_\odot$	$\lg \mathfrak{M}_0$	$\lg R_\odot$	$\lg L_\odot$	$\lg \mathfrak{M}_0$	$\lg R_\odot$	$\lg L_\odot$	$\lg \mathfrak{M}_0$	$\lg R_\odot$	$\lg L_\odot$	$\lg \mathfrak{M}_0$	$\lg R_\odot$	$\lg L_\odot$
$\lg \varepsilon_0 = -124$															
0.01	0.15130	—0.16977	0.15130	0.22380	—0.11926	0.51474	0.54551	0.10571	2.12749	0.74678	0.24647	3.13648	1.18344	0.55185	5.32548
0.1	0.36373	—0.09514	0.36373	0.53801	—0.2170	1.2380	1.31	0.768	5.12	1.791	1.092	7.52	2.84498	1.83040	12.80241
0.2	0.37424	—0.14831	0.37424	0.55356	—0.27372	1.27319	1.35	0.831	5.27	1.85	1.180	7.21	2.92721	1.93372	13.17244
0.4	0.29292	—0.15560	0.29292	2.43327	—0.2540	0.9980	1.057	0.692	4.12	1.447	0.925	6.07	2.29112	1.55304	10.31004
$\lg \varepsilon_0 = -123$															
0.01	0.16455	—0.09995	0.16455	0.24339	—0.04481	0.55980	0.59327	0.19988	2.31375	0.81217	0.35296	3.41111	1.28706	0.68531	5.79177
0.1	0.37698	—0.16517	0.37698	—	—	—	—	—	—	—	—	—	2.94859	1.96361	13.26866
0.2	0.38749	—0.21834	0.38749	0.57316	—0.34818	1.31827	—	—	—	—	—	—	3.03083	2.06695	13.63874
0.4	0.30617	—0.22563	0.30617	—	—	—	—	—	—	—	—	—	2.39474	1.68626	10.77633

The results of the calculations are given in Table 1.

The variation of log L with log R for Z = 0.01 is given in Fig. 1; it is practically a linear relationship. The deviation from a straight line is given by the term 8 log R_f which is small by comparison with 107 log $\mathfrak{M}_1$. For each value of γ, the line Z = const represents an evolutionary sequence for a homogeneous model with a definite initial chemical composition (which is characterized by the value of Z). The mass of the star and the hydrogen-to-helium ratio changes along this sequence. The variation of the latter is governed by the occurrence of nuclear reactions in the central regions of the star.

In order to estimate the degree to which the theoretical sequences under consideration agree with those observed, it is necessary to plot them on the Hertzsprung-Russell diagram. To obtain the values of $\mathfrak{M}_0$ and R_0 to be used in Expressions (2), we use the two theoretical relationships between the main stellar parameters which were obtained in the construction of the model (Eqs. (6), (10) and (35), (35') of [1]). The calculations were carried out for two values of ϵ_0, the energy-generation parameter [Eq. (1)]: $\epsilon_0 = 2 \cdot 10^{-123}$ and $\epsilon_0 = 2 \cdot 10^{-124}$ with n = 17. The results of the calculation are given in Table 2.

The dependence of log R_0 and log $\mathfrak{M}_0$ on γ is given in Figs. 2 and 3. From Figs. 2 and 3 it is seen that for each value of Z, $\mathfrak{M}_0$ and R_0 increase with γ and tend to infinity for $\gamma \sim 5$.

Let us investigate the question of the existence of a limiting value of γ more fully.

Let us obtain the solution of Eqs. (3), (4) and (5) in a general form:

$$\mathfrak{M}^a R^b = \alpha(X, Z); \qquad (11)$$

$$\mathfrak{M}^c R^d = \beta(X, Z); \qquad (12)$$

For our model:

$$a = 5.625 - \gamma$$
$$b = -0.875$$
$$c = n + 2 - \gamma$$
$$d = -n - 3$$

For Schwarzschild's model:

$$\left.\begin{aligned} a &= 5.75 - \gamma \\ b &= -1.25 \\ &n + 2 - \gamma \\ &-n - 3 \end{aligned}\right\} \qquad (13)$$

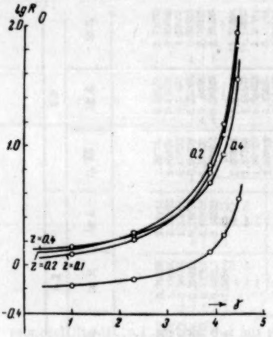

Fig. 2. Log R_0 as a function of γ for various values of Z (the lower curve corresponds to Z = 0.01).

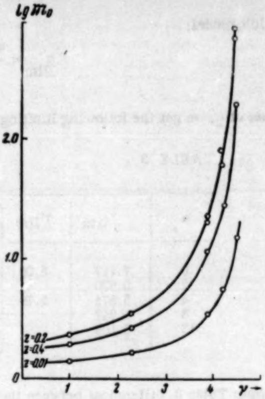

Fig. 3. Log $\mathfrak{M}_0$ as a function of γ for various values of Z.

$$\left.\begin{array}{ll}\alpha = 1.145\cdot 10^{-2} f_1(X, Z) & \alpha = 4.12\cdot 10^{-3} f_1(X, Z) \\ \beta = 5.984\cdot 10^{-8} f_2(X, Z) & \beta = 4.24\cdot 10^{-7} f_2(X, Z)\end{array}\right\}, \tag{14}$$

where

$$\left.\begin{array}{l} f_1(X, Z) = (1+X)\,Z\left(\frac{1+5X-Z}{2}\right)^{7.5} \\ f_2(X, Z) = \left[\frac{1}{2}(1+5X-Z)\right]^n / XZ \end{array}\right\}. \tag{15}$$

After taking logarithms, we obtain the solution of the System of Eqs. (11) and (12) in the form

$$\lg \mathfrak{M} = \lg \mathfrak{M}^* / D;$$

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix}; \ \lg \mathfrak{M}^* = \begin{vmatrix} \lg\alpha & b \\ \lg\beta & d \end{vmatrix} \tag{16}$$

$$\lg R = \frac{\lg R^* + (c-a)\lg \mathfrak{M}}{b-d}, \qquad \lg R^* = \lg\alpha - \lg\beta. \tag{17}$$

The limiting value of γ is obtained from the condition D = 0. For our model:

$$\gamma_{\lim} = \frac{38n+121}{8n+15}; \tag{18}$$

for Schwarzschild's model:

$$\gamma'_{\text{lim}} = \frac{37n + 120}{8n + 15}. \tag{19}$$

For various values of n, we get the following limiting values:

TABLE 3

n	γ_{lim}	γ'_{lim}	n	γ_{lim}	γ'_{lim}
0	7.117	8.00	15	5.044	5.000
2	5.970		17	5.013	4.960
4	5.571	5.70	18	5.000	
8	5.247		30	4.907	
12	5.106		∞	4.750	4.63

As can be seen from Table 3, differences between the values of a and b for the two models, i.e., the differences between the models (in the law of absorption), have only a small influence on the limiting values of γ. The energy-generation law plays the main role. For the carbon cycle ($n \approx 15$-18) $\gamma_{\text{lim}} \sim 5$, and for the proton reaction ($n \sim 4$) $\gamma_{\text{lim}} = 5.6$.

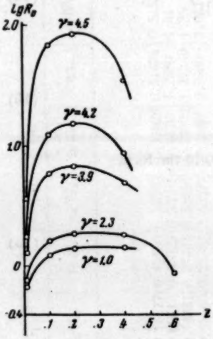

Fig. 4. Log R_0 as a function of Z for several values of γ.

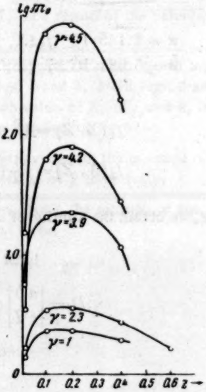

Fig. 5. Log $\mathfrak{M}_0$ as a function of Z for several values of γ.

The relations between log R_0 and Z and between log $\mathfrak{M}_0$ and Z are given in Figs. 4 and 5. As was already pointed out earlier in [3], for each value of γ there exist certain maximum possible values of $\mathfrak{M}_0$ and R_0. For a given value of γ, these maximum values of $\mathfrak{M}_0$ and R_0, in general, do not occur at the same value of Z; the exception to this is the case $\gamma = 3.9$. This means that for a given value of γ the maximum value of the mass (or luminosity) and the maximum value of the radius cannot, as a rule, describe actual stellar sequences for which there exist two-parameter relationships(mass-luminosity and mass-radius)of the type $L \sim \mathfrak{M}^{\gamma}$; $R \sim \mathfrak{M}^{\delta}$

TABLE 4

Z	Y	1.0			2.3			3.9			4.2			4.5		
	X	lg 𝔐	lg R	lg L	lg 𝔐	lg R	lg L	lg 𝔐	lg R	lg L	lg 𝔐	lg R	lg L	lg 𝔐	lg R	lg L
lg e_0 = −123																
0.01	0.99	0.16455	−0.09995	0.16455	0.24339	−0.04481	0.55980	0.59327	0.19988	2.31375	0.81217	0.35296	3.41111	1.28706	0.68531	5.79177
	0.9	0.11363	−0.12602	0.11363	0.16806	−0.08795	0.38654	0.41153	0.08194	1.60496	0.56084	0.18666	2.35572	0.88874	0.41621	3.99933
	0.8	0.05055	−0.15628	0.05055	0.08140	−0.13787	0.18699	0.19689	−0.05698	0.76787	0.27132	−0.00494	1.13954	0.42992	0.10621	1.93464
	0.7	−0.00872	−0.18919	−0.00872	−0.01241	−0.19212	−0.02969	−0.03347	−0.20640	−0.13054	−0.04304	−0.20924	−0.18077	−0.06828	−0.22969	− 0.30726
	0.6	−0.07726	−0.22496	−0.07726	−0.11430	−0.25086	−0.26289	−0.28197	−0.36777	−1.09969	−0.38132	−0.43764	−1.60156	−0.60439	−0.59139	− 2.71975
	0.5	−0.15207	−0.26395	−0.15207	−0.22158	−0.31256	−0.50963	−0.55190	−0.54334	−2.15241	−0.85053	−0.68254	−3.15223	−1.18952	−0.98929	− 5.35284
	0.4	—	—	—	—	—	—	−1.53926	−1.19089	−6.00306	−2.118	−1.588	−8.86	—	—	—
0.1	0.9	0.37698	0.16517	0.37698	—	—	—	—	—	—	—	—	—	2.94859	1.96361	13.26866
	0.8	0.31676	0.13422	0.31676	—	—	—	—	—	—	—	—	—	2.47758	1.64537	11.14907
	0.7	0.25228	0.10102	0.25228	—	—	—	—	—	—	—	—	—	1.97320	1.30453	8.87941
	0.6	0.18234	0.06481	0.18234	—	—	—	—	—	—	—	—	—	1.42612	0.93464	6.41754
	0.5	0.10641	0.02521	0.10641	—	—	—	—	—	—	—	—	—	0.83220	0.53278	3.74491
0.2	0.8	0.38749	0.21834	0.38749	0.57316	0.34818	1.31827	—	—	—	—	—	—	3.03083	2.06695	13.63874
	0.7	0.32202	0.18468	0.32202	0.47632	0.29259	1.09554	—	—	—	—	—	—	2.51872	1.72094	11.33429
	0.6	0.27801	0.16689	0.27801	0.37144	0.23222	0.85431	—	—	—	—	—	—	1.96413	1.34607	8.83859
	0.5	0.21837	0.13886	0.21837	0.25730	0.16607	0.59179	—	—	—	—	—	—	1.36053	0.93762	6.12239
	0.4	—	—	—	0.13240	0.09538	0.30452									
0.4	0.6	0.30617	0.22563	0.30617	—	—	—	—	—	—	—	—	—	2.39474	1.68626	10.77633
	0.5	0.22905	0.18554	0.22905	—	—	—	—	—	—	—	—	—	1.79151	1.27832	8.06179
	0.4	0.14454	0.14117	0.14454	—	—	—	—	—	—	—	—	—	1.13048	0.83068	5.08716
	0.3	0.05045	0.09137	0.05045	—	—	—	—	—	—	—	—	—	0.39453	0.33150	1.77539
lg e_0 = −124																
0.01	0.99	0.15130	−0.16997	0.15130												
	0.9	0.10038	−0.19604	0.10038												
	0.8	0.04180	−0.22630	0.04180												
	0.7	−0.02197	−0.25921	−0.02197												
	0.6	−0.09051	−0.29498	−0.09051												
	0.5	−0.16532	−0.33397	−0.16532												
0.1	0.9	0.36373	0.09514	0.36373												
	0.8	0.30351	0.06419	0.30351												
	0.7	0.23903	0.03099	0.23903												
	0.6	0.16909	−0.00522	0.16909												
	0.5	0.09316	−0.04482	0.09316												
0.2	0.8	0.37424	0.14381	0.37424												
	0.7	0.30877	0.11465	0.30877												
	0.6	0.26476	0.09686	0.26476												
	0.5	0.20512	0.06883	0.20512												
0.4	0.6	0.29292	0.15560	0.29292												
	0.5	0.21580	0.11551	0.21580												
	0.4	0.13129	0.07114	0.13129												
	0.3	0.03765	0.02134	0.03765												

TABLE 5

Y		1.0				2.3				3.9				4.2				4.5			
Z	X	M_b	$\lg T_{eff}$	M_V	Sp	M_b	$\lg T_{eff}$	M_V	Sp	M_b	$\lg T_{eff}$	M_V	Sp	M_b	$\lg T_{eff}$	M_V	Sp	M_b	$\lg T_{eff}$	M_V	Sp
$\lg e_*=-123$																					
0.01	0.99	4.4	3.83	4.4	F5	3.4	3.90	3.6	A7	−1.0	4.22	1.3	B6	− 3.7	4.41	− 1.0	B0	− 9.7	4.85	− 6.2	O6
	0.9	4.5	3.83	4.5	F5	3.8	3.88	3.9	A9	+0.8	4.10	1.8	B8	− 1.1	4.24	+ 0.7	B4	− 5.2	4.54	− 2.2	O8
	0.8	4.7	3.83	4.7	F5	4.3	3.86	4.3	F1	2.9	3.96	3.3	A4	2.0	4.02	2.7	A0	0.0	4.17	1.5	B6
	0.7	4.8	3.83	4.8	F5	4.8	3.83	4.8	F5	5.1	3.81	5.1	F6	5.2	3.80	5.2	F7	5.6	3.77	5.7	G1
	0.6	5.0	3.83	5.0	F5	5.5	3.79	5.5	F8	7.5	3.65	7.7	K3	8.8	3.56	10.7	M0	11.6	3.36	15.6	M5
	0.5	5.2	3.83	5.2	F5	6.1	3.77	6.2	G0	10.2	3.47	12.9	M8	12.7	3.29	—	—	18.2	2.85	—	—
	0.2	—	—	—	—	—	—	—	—	19.8	2.84	—	—		—	—	—	—	—	—	—
	0.9	3.9	3.75	4.0	G2	—	—	—	—	—	—	—	—					−28.3	6.07	—	—
	0.8	4.0	3.75	4.1	G2	—	—	—	—	—	—	—	—					−23.1	5.71	—	—
	0.7	4.2	3.75	4.3	G2	—	—	—	—	—	—	—	—					−17.4	5.31	−12.9	O2
	0.6	4.3	3.75	4.4	G2	—	—	—	—	—	—	—	—					−11.2	4.87	− 7.6	O5
	0.5	4.5	3.75	4.6	G2	—	—	—	—	—	—	—	—					− 4.6	4.41	− 1.9	B0
0.2	0.8	3.8	3.73	3.9	G5	1.50	3.90	1.7	A8	—	—	—	—					−29.3	6.12	—	—
	0.7	4.0	3.73	4.1	G5	2.06	3.86	2.1	F0	—	—	—	—					−23.5	5.71	—	—
	0.6	4.4	3.73	4.2	G5	2.69	3.83	3.0	F3	—	—	—	—					−17.3	5.28	−13.0	O2
	0.5	4.3	3.72	4.4	G5	3.32	3.81	3.3	F7	—	—	—	—					−10.5	4.80	− 6.9	O6
	0.4	—	—	—	—	4.04	3.77	4.1	G1	—	—	—	—					—	—	—	—
0.4	0.6	4.0	3.71	4.1	G8	—	—	—	—	—	—	—	—					−22.1	5.59	—	—
	0.5	4.2	3.71	4.3	G8	—	—	—	—	—	—	—	—					−15.4	5.12	−11.2	O3
	0.4	4.4	3.71	4.5	G8	—	—	—	—	—	—	—	—					− 7.9	4.59	− 4.8	O8
	0.3	4.7	3.70	4.8	G8	—	—	—	—	—	—	—	—					+ 0.4	4.09	+ 1.4	B8
$\lg e_*=-124$																					
0.01	0.99	4.5	3.86	4.6	F1									− 3.0	4.40	− 0.4	B0				
	0.9	4.6	3.86	4.7	F1									− 0.4	4.22	1.3	B4				
	0.8	4.7	3.86	4.8	F1									2.6	4.02	3.3	A0				
	0.7	4.8	3.87	4.9	F0									5.9	3.78	5.9	F9				
	0.6	5.0	3.87	5.1	F0									9.5	3.54	11.2	M1				
	0.5	5.2	3.87	5.2	F0									13.4	3.27	—	—				
0.1	0.9	4.0	3.77	4.1	G0																
	0.8	4.1	3.78	4.1	F9																
	0.7	4.3	3.78	4.3	F9																
	0.6	4.4	3.78	4.4	F9																
	0.5	4.6	3.78	4.6	F9																
0.2	0.8	4.0	3.75	4.1	G2																
	0.7	4.1	3.75	4.2	G2																
	0.6	4.2	3.75	4.3	G2																
	0.5	4.3	3.76	4.4	G1																
0.4	0.6	4.2	3.72	4.3	G6																
	0.5	4.3	3.73	4.4	G5																
	0.4	4.5	3.73	4.6	G5																
	0.3	4.7	3.74	4.8	G4																

The case $\gamma = 3.9$ is an exception and here the maximum values of $\mathfrak{M}_0$ and R_0 are attained for $Z \sim 0.2$. Consequently, in the case of the models under consideration the evolutionary sequence with $\gamma = 3.9$ can be used to describe those stellar sequences that can be characterized by two-parameter relationships (mass-luminosity and mass-radius).

As was shown in [1, 2, 3], in this case the exponent in the radius-mass relationship is found to be $\delta \approx 0.7$, i.e., we have the case of the actual main sequence.

It is obvious that if we want to discuss the possibility of applying the sequences obtained above for various values of γ, then, with the exception of $\gamma = 3.9$, we can only consider those stellar groups for which there are no two-parameter relationships: mass-luminosity and mass-radius. Subdwarfs and subgiants are stars of this type.

TABLE 6

X	Y	$A(X; 0.1)$	$B(X; 0,1)$	$\lg T_{\text{eff}}$
0.9	0.0	3.79	0.377	3.92
0.8	0.1	3.78	0.317	3.89
0.7	0.2	3.77	0.253	3.85
0.6	0.3	3.77	0.195	3.83
0.5	0.4	3.76	0.109	3.80
0.4	0.5	3.76	0.023	3.77
0.3	0.6	3.75	—0.071	3.73
0.2	0.7	3.74	—0.167	3.68
0.1	0.8	3.74	—0.412	3.60

Tables 4 and 5 give the quantities appearing in Table 1 transformed to the coordinates of the Hertzsprung-Russell diagram by means of the usual expressions

$$M_b = 4.8 - 2.5 \log L, \tag{20}$$

$$\log T_{\text{eff}} = 4.22 - 0.1\, M_b - 0.5 \log R. \tag{21}$$

Substituting into (20) and (21) the values given by (16) and (17), we get

$$\lg T_{\text{eff}} = A(X, Z) + \frac{\gamma - 2\frac{c-a}{b-d}}{D} B(X, Z). \tag{22}$$

$$M_b = 4.8 - \frac{10\gamma}{D} B(X, Z), \tag{23}$$

where

$$A(X, Z) = 3.74 - \frac{\lg \alpha - \lg \beta}{2(b-d)}; \tag{24}$$

$$B(X, Z) = \frac{1}{4} \begin{vmatrix} \lg \alpha & b \\ \lg \beta & d \end{vmatrix}. \tag{25}$$

Eliminating B(X, Z), we get

$$\lg T_{\text{eff}} = A(X, Z) - \frac{1}{10\gamma}\left(\gamma - 2\frac{c-a}{b-d}\right)(M_b - 4.8). \tag{26}$$

Values of T_{eff} are given in Table 6 for $n = 17$, $\gamma = 2.3$, $Z = 0.1$, and $\log \epsilon_0 = 123$.

Since $A(X, Z)$ remains practically constant, Eq. (26) gives a linear relationship between the bolometric magnitude and the logarithm of the effective temperature. The variation of $A(X, Z)$ becomes important only for small values of γ. In fact, for $\gamma = 2\frac{c-a}{b-d}$, $\log T_{\text{eff}} = A(X, Z)$. For our model, this corresponds to $\gamma =$ $= 1.396$; for Schwarzschild's model, to $\gamma = 1.412$.

The change according to the models considered therefore takes place in the plane (M_b, $\log T_{\text{eff}}$) along straight lines (with a negligible departure from linearity). That this is in fact so can be seen from Fig. 6, in which the results of Tables 4 and 5 have been plotted for $\log \epsilon_0 = -123$. For a given value of Z, the straight lines corresponding to various values of γ intersect at the point $M_b = 4.8$, i.e., $B(X, Z) = 0$ (Fig. 6). The slope of these straight lines is positive for small γ and negative for large γ. Consequently, cases are possible when, in the course of evolution with decreasing mass the spectrum remains unchanged or even changes to that of an earlier type. In Fig. 6, the dotted curves give the envelopes of the straight lines γ = const (for $\gamma = 2.3$ and $\gamma = 1.0$) for different values of Z. Fig. 7 gives the displacement of these envelopes as a function of $\log \epsilon_0$. The calculations were also carried out for $\log \epsilon_0 = -122$ and -127, in addition to those given in Tables 4 and 5

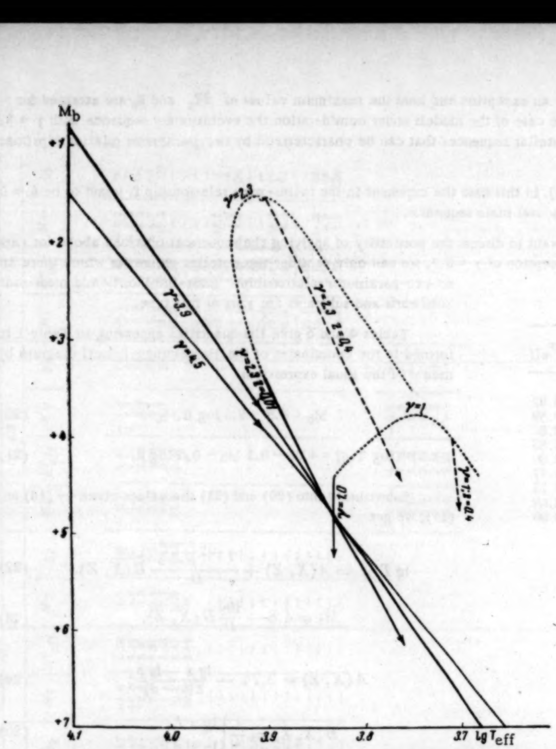

Fig. 6. The evolutionary paths Z = const in the (M_b, log T_{eff}) plane for various values of γ; full line – Z = 0.01; dashed line – Z = 0.4; dotted curve – the envelope γ = constant for various values of Z(log $\epsilon_0 = -123$).

for log $\epsilon_0 = -123$ and -124. The effect of changing ϵ_0 can be easily estimated. Since B (X, Z) ~ ϵ, a variation of this quantity results in a parallel displacement of the evolutionary paths approximately at right angles to the main sequence. As log ϵ_0 increases, T_{eff} increases.

In Fig. 8 the derived evolutionary paths have been plotted on the Hertzsprung-Russell diagram. The curves Z ≤ 0.01 for γ = 1.0 and γ = 2.3 lie in the region of subdwarfs. Since the model under consideration gives Z = 0.16 in the case of the Sun, the structure and evolution of subdwarfs can be explained with the help of the semi-convective model only on the assumption that the relative abundance of heavy-element mixtures in them is considerably less (by a factor of almost 20) than that in the main-sequence stars. Evolution in the case of such stars must then proceed in the downward direction from the main sequence: for γ = 2.3 in the direction of the later spectral types, while for γ = 1 in the direction of the earlier spectral types (Fig. 8). The limiting mass of such subdwarfs is $\mathfrak{M}$ = 1.75 ⊙ for γ = 2.3 and $\mathfrak{M}$ = 1.46 ⊙ for γ = 1. Inasmuch as the rate of evolution is the greater the greater the mass and luminosity of the stars, the separation between the main sequence and the subdwarf sequence must be greater in the region of the earlier spectral types than it is in the region of the later spectral types; in general, this is in fact the case. It is interesting to note that among the subdwarfs whose masses are known there is not one subdwarf with a mass exceeding these limits (see [4]). As regards the subgiants, our curves Z > 0.4 for γ = 2.3 and γ = 1 partially enclose the region occupied by these stars in the Hertzsprung-Russell diagram. It appears that in this case we are dealing with stars which, as was earlier discussed

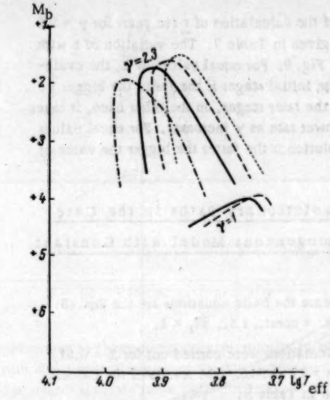

Fig. 7. The position of the curves γ = const as a function of ϵ_0; full line – log ϵ_0 = −124, dashed line – log ϵ_0 = −123, dotted line – log ϵ_0 = −122, dot-dashed line – log ϵ_0 = −127.

TABLE 7

γ	2.3	2.3	4.5
X \ Z	0.01	0.2	0.01
0.90	$2.63 \cdot 10^{10}$	—	$1.97 \cdot 10^{6}$
0.8	$5.99 \cdot 10^{10}$	0.00	$8.08 \cdot 10^{7}$
0.7	$1.06 \cdot 10^{11}$	$1.05 \cdot 10^{10}$	$4.39 \cdot 10^{9}$
0.6	$1.64 \cdot 10^{11}$	$2.39 \cdot 10^{10}$	$3.30 \cdot 10^{11}$
0.5	$2.44 \cdot 10^{11}$	$4.28 \cdot 10^{10}$	$3.67 \cdot 10^{14}$
0.4		$7.16 \cdot 10^{10}$	

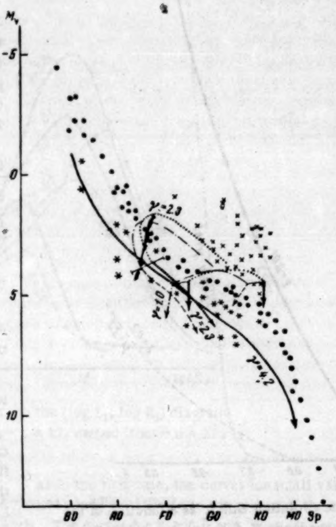

Fig. 8. The evolutionary paths for Z = 0.01 and the curves γ = const for log ϵ_0 = −123 and −124 plotted on the Hertzsprung-Russell diagram; ● – main-sequence stars, • – subdwarfs, + – subgiants; full line – the curves Z = 0.01 for log ϵ_0 = −123, dashed lines – the curves Z = 0.01 for log ϵ_0 = −124, dotted lines – the curves γ = const for log ϵ_0= −123, dot-dashed lines – the curves γ = const for log ϵ_0 = −124.

by one of the authors [5], were originally formed with a considerably higher (by a factor of 4-5) abundance of the heavy elements than that in the case of the main-sequence stars. The evolution of these stars can take place with constant as well as variable mass: as the stars evolve, some of the subgiants move away from the main sequence and some, on the other hand, move towards it. As a result of this, the dispersion in luminosity shown by the region of the subgiants in the Hertzsprung-Russell diagram is considerable, while the spectral range is very small (for details, see [5]). It should be noted that by suitably altering the value of ϵ_0, we can obtain a displacement of the evolutionary paths for any value of γ such that they will cover the region of the diagram occupied by the subgiants and the subdwarfs ($-118 >$ log $\epsilon_0 > -127$). However, such a variation in ϵ_0 can hardly be justified on physical grounds.

Let us investigate the evolution times for our curves:

$$t = \frac{\eta \mathfrak{M}_0}{L_0} \sigma, \tag{27}$$

where $\eta = 6.4 \cdot 10^{18}$ ergs.

$$d\sigma = -\mathfrak{M}_1^{1-\gamma}(X, Z)\, dX. \tag{28}$$

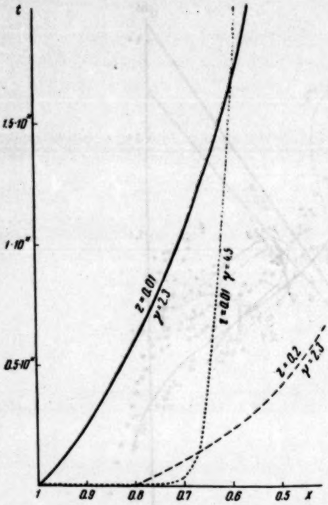

Fig. 9. The evolution time t as a function of the hydrogen abundance X; full line – $\gamma = 2.3$, $Z = 0.01$; dashed line – $\gamma = 2.3$, $Z = 0.2$; dotted line – $\gamma = 4.5$, $Z = 0.01$.

The results of the calculation of t (in years for $\gamma = 2.3$ and 4.5) are given in Table 7. The variation of t with X is given in Fig. 9. For equal values of Z, the evolution during the initial stages is the faster the bigger γ, while during the later stages, on the other hand, it takes place at a slower rate as γ increases. For equal values of γ, the evolution is the faster the bigger the value of Z.

2. The Evolutionary Paths in the Case of the Homogeneous Model with Constant Mass

In this case the basic equations are the Eqs. (3) and (4) for $\mathfrak{M}$ = const., i.e., $\mathfrak{M}_1 = 1$.

The calculations were carried out for Z = 0.01, 0.1, 0.5 and n = 17 and 15. The results of the calculations are given in Table 8.

The evolutionary paths Z = const on the ($\log L_1$, $\log R_1$) diagram are given in Fig. 10.

As X decreases, the radius of the star at first increases, then it reaches a certain maximum value and finally decreases. The change in R_1 for n = 17 is greater than that for n = 15. During the course of the evolution, L_1 continues to increase (up to X = 0.001).

The evolutionary paths for the homogeneous model with a constant mass equal to 1⊙, 2⊙, 5⊙ and 10⊙ are given in Fig. 11.

TABLE 8

X	n = 17		n = 15	
	lg L_1	lg R_1	lg L_1	lg R_1
$Z = 0.01$				
0.99	0.000	0.000	0.000	0.000
0.90	0.201	0.009	0.203	0.007
0.80	0.441	0.020	0.444	0.016
0.70	0.697	0.032	0.708	0.025
0.60	0.957	0.045	0.986	0.035
0.40	1.188	0.078	1.27	0.057
0.20	1.31	0.117	1.42	0.079
0.01	1.60	0.104	1.79	0.049
0.001	1.78	0.053	2.02	0.010
$Z = 0.1$				
0.90	0.000	0.000	0.000	0.000
0.60	0.781	0.035	0.787	0.029
0.20	1.31	0.093	1.37	0.072
0.10	1.40	0.104	1.47	0.079
0.01	1.59	0.083	1.72	0.045
0.001	1.78	0.033	1.94	0.010
0.0001	1.98	0.010	2.18	0.009
$Z = 0.50$				
0.50	0.000	0.000	0.000	0.000
0.40	0.340	0.016	0.360	0.013
0.30	0.760	0.033	0.766	0.027
0.25	0.970	0.041	0.982	0.034
0.10	1.19	0.068	1.22	0.049
0.01	1.35	0.049	1.42	0.017
0.001	1.50	0.010	1.63	0.010
0.0001	1.67	0.009	1.86	0.009

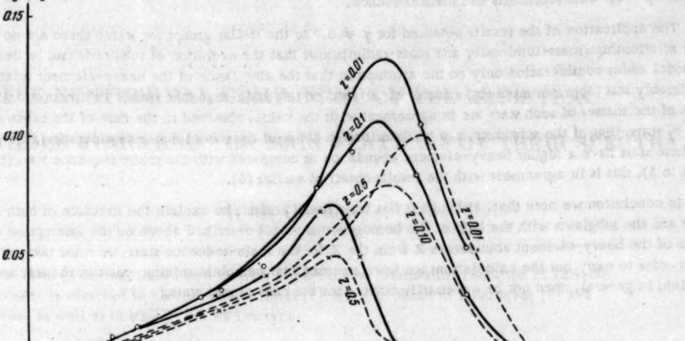

Fig. 10. The evolutionary paths Z = const on the (log L_1, log R_1) diagram for the case of constant masses; full lines – n = 17, dashed lines – n = 15.

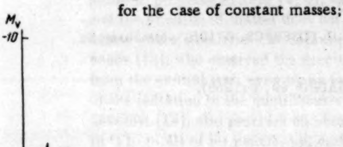

Fig. 11. The evolutionary paths Z = const in the Hertzsprung-Russell diagram for several values of the mass; (the symbols are the same as in Fig. 8); full lines – Z = 0.01; dashed lines – Z = 0.5.

As in the first case, the curves for small values of Z (0.01) and $\mathfrak{M} \sim 1\odot$ lie in the region of the subdwarfs. The curves for $Z \geq 0.5$ and $\mathfrak{M} = 2\odot$ connect the region of the subgiants with the main sequence.

3. Summary

We have investigated all the possibilities for applying the homogeneous model of a gaseous star with a convective core and the evolution predicted from this model in the cases of a constant and a variable mass.

The various possible laws for the variation of the mass [various values of γ in Eq. (5)] have been investigated. There exists for each model a certain limiting value of γ which depends on the constant ϵ_0 in the energy-generation law and which is practically independent of the model itself. For each possible value of γ there exist certain maximum values of $\mathfrak{M}_0$ and R_0, whose absolute values depend on the form of the model adopted. However, these maximum values of the mass and luminosity are attained at different values of Z for different values of γ, and for a given value of γ these maxima do not occur at the same value of Z; the exception to this is the case $\gamma = 3.9$. This means that the evolutionary sequences investigated cannot describe the real stellar sequences for which there exist the two-parameter relationships, mass-luminosity and mass-radius. The only exception is the case $\gamma = 3.9$, in which the maximum values of $\mathfrak{M}_0$ and R_0 are attained for one and the same value of Z (for our model $Z = 0.2$). As was shown earlier [1, 2, 3], the theoretical curve for $\gamma = 3.9$ well represents the main sequence.

The application of the results obtained for $\gamma \neq 3.9$ to the stellar groups for which there are no two-parameter relationships(mass-luminosity and mass-radius)shows that the evolution of subdwarfs can be described by the model under consideration only on the assumption that the abundance of the heavy-element mixture, Z, is considerably less (approximately by a factor of 20) than in the main-sequence stars. The theoretically possible values of the masses of such stars are in agreement with the values observed in the case of the subdwarfs. Another group of stars, that of the subgiants, can be described in terms of the model under consideration, if it is assumed that these stars have a higher heavy-element abundance as compared with the main-sequence stars (in the ratio of 4-5 to 1); this is in agreement with the results obtained earlier [5].

In conclusion we note that, although it has been found possible to explain the structure of both the subdwarfs and the subgiants with the help of the homogeneous model described above on the assumption of a departure of the heavy-element abundances Z from the Z for the main-sequence stars, we must take into account that in order to carry out the calculations we have assumed that complete mixing exists in the star and this assumption, in general, need not be necessarily correct for the case of real stars.

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THE NONTHERMAL COMPONENT OF THE RADIATION OF UNSTABLE STARS AND THE MAIN FEATURES OF THEIR SPECTRA

I. M. Gordon

The author has shown in a number of papers that nonthermal radiation of variable intensity is observed in chromospheric flares [1], the explosions of supernovae [2] and novae, as well as in stars of the RVTau type.

V. A. Ambartsumian [3] and the author [4], independently, have also established that the luminosity variations in UVCet and TTau stars are nonthermal in origin. The author has proposed that "light-emitting" electrons are the source of the nonthermal radiation in all cases. One of the main deductions of the new theory of supernova explosions, proposed in 1953 [2, 5], was that the production of "light-emitting" electrons and the ejection of matter does not occur only once, but must also be observed for a considerable time after the explosion. This aspect of the theory was confirmed by Baade [18], who observed the ejection of condensations of "light-emitting" electrons from the central star, even many centuries after the explosion. The nonthermal nature of the radiation in the continuous-emission "grains," discovered on the sun by A. B. Severnyi [19], also provides an observational confirmation of the conclusions presented in [1]. In all of his papers, the author pointed out the necessity for carrying out measurements of the polarization of stellar radiation in order to check the hypothesis of the synchrotron nature of the radiation and showed that considerable polarization may be present when the luminosity varies.

The present work, which is a continuation of the author's work mentioned above and also that of references [25, 22], has two aims:

a) To show that a series of observational data confirm the presence of a nonthermal component in the radiation of novae.

b) To investigate the influence of the nonthermal radiation on the important features of unstable-star spectra and to compare the results obtained with observation.

1. The deduction, obtained in references [25, 22], that the rapid luminosity variations of N. Lac 1950 are of nonthermal origin,is confirmed. The rapid luminosity variations of N. Her 1934 many years after the explosion are also nonthermal in origin and show that the activity, of the same type as is observed in supernovae [2], also occurs in novae. The observations of Herzog [39] that soon after the maximum,N. Her 1934 was also an eclipsing binary disprove the theory according to which the increase of luminosity is governed by an expansion of the photosphere. The radius of the orbit is $2 \cdot 10^{10}$ cm, while the radius of the photosphere before the sudden luminosity decrease in April was $\sim 2.1 \cdot 10^{11}$ cm, i.e., an order of magnitude greater than the radius of the orbit. Thus, if the radiation were of thermal origin, the effect of the eclipse could not have been observed.

2. The deduction that the effect of the "flooding" by the nonthermal emission of the spectra of novae, discovered first by V. A. Ambartsumian for the TTau stars

[3] and then by the author in the case of novae [5], explains a number of important features in their spectra, is confirmed. It is also shown that this effect explains the variation of the abundance of iron atoms in the atmosphere of U Mon without any change of the ionization, as observed by H. Abt [48].

3. It is shown that intense nonthermal radiation can lead to a redistribution of atoms in the excited states [25, 22] and a transformation of absorption into emission lines. Under certain conditions the induced transitions begin to dominate over the spontaneous ones. The way in which the stellar spectra depend on the value of the "coefficient of nonthermal excess" (CNE), introduced in [25], is investigated.

1) The nonthermal excess shortens the lifetime of the excited states, i.e., a strengthening of the forbidden lines becomes possible.

2) The nonthermal excess changes the natural width of the lines, as a result of which the width varies with height in the atmosphere — a possible reason for the existence of complex line profiles.

3) For large values of CNE the emission lines of unstable stars have the character of negative absorption lines. Therefore, they can arise in a fairly thin effective layer. The height of the lines can depend on the composition of the continuous radiation and the distribution of its intensity with height. In the presence of a gradient in the velocity or magnetic field, a change in the value of the CNE can lead to apparent variations of the radial velocity and the magnetic field, as well as of temperature and spectral class. The same phenomenon can also be observed in the case of absorption lines.

4) It appears that the explanation of the simultaneous variations of the intensity and velocity displacement of forbidden and allowed lines in the spectra of a number of novae is the change in the composition of the nonthermal radiation; these simultaneous variations cannot be explained by changes in the rate of ejection.

5) The nonthermal excess and its variation in time can be the explanation for the anomaly in the Balmer decrement and its time variations in the spectra of novae and other nonstationary stars.

6) It is pointed out that under certain conditions polarization of the nonthermal radiation can occur as the polarization of the absorption and emission lines in the spectra of unstable stars.

It is shown that the introduction of the quantity CNE into the theory of stellar spectra creates a new degree of freedom, governing their basic properties. It is pointed out that investigations of the rapid variations of the luminosity and color, as well as the measurement of the polarization of the spectra of unstable stars, are required.

INTRODUCTION

Investigations carried out in the last few years have shown that various nonstationary processes in stars give rise to nonthermal optical radiation. It has been shown that the radiation with a continuous spectrum from chromospheric flares, observed in white light on the sun [1], the radiation of supernovae [2], as well as of UVCet and T Tau stars [3,4], has a nonthermal nature.

In these papers [1, 2, 4, 5] it was proposed that the nonthermal radiation in the spectra of nonstationary stars is the synchrotron radiation emitted by relativistic electrons moving in magnetic fields and it was pointed out that investigations of the polarization of the light could provide an unambiguous verification of this hypothesis.*

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* Note added in proof. The polarization of the light emitted during luminosity variations in unstable stars, predicted by us [4], was recently discovered at the Lick Observatory during the study of the polarization in the ultraviolet region of the spectrum of a star of the type T Tau, NX Mon [51]. Rapid random variations of the luminosity by 1^m over a period of 1 minute and an unusually high degree of polarization, 35-55%, were observed.

These observations provide a confirmation of our model which establishes the relation between the occurrence of nonstationary processes and the production of cosmic rays [4, 40]. See [22] for greater detail.

The hypothesis of "light-emitting" electrons was first confirmed when I. S. Shklovskii showed that thermal emission was inadequate to account for the optical radiation from the Crab Nebula M1 [6].

The author pointed out that an investigation of polarization in the case of M1 could provide an unambiguous confirmation of the above hypothesis [1,7]. Soon afterwards, a number of investigations established that the radiation from M1 was extensively polarized and, therefore, fully confirmed the hypothesis of "light-emitting" electrons [8-12].

The confirmation of the theory of "light-emitting" electrons as applied to the Crab Nebula is the starting point for the theory of cosmic-ray origins being developed at present. V. L. Ginzburg[13], I. S. Shklovskii [14], and S. B. Pikel'ner [15] have shown that the use of radioastronomical observations is of the greatest importance in the development of this theory.

In particular, first of all the author [2, 5, 16] and then, independently, J. Oort [12] showed that the production of relativistic particles after a supernova explosion occurs in the atmosphere of the central star and not in the expanding envelope. The latest investigations of S. B. Pikel'ner [17] also confirm this hypothesis. The new theory of supernovae explosions developed by the author in 1953, according to which the increase of the luminosity is determined by the production of a large number of "light-emitting" electrons in the atmosphere of the star, has therefore been placed on a firm basis,since the "radioelectrons" must emit intense optical radiation in traversing the atmosphere of the star where the magnetic field is several orders of magnitude greater than that in the nebula.

One of the main deductions of this theory was that the production of relativistic particles and the ejection of matter occur not just once, but continuously,and continue for a considerable time after the explosion [2].

With respect to these decisive features, the theory has been confirmed by the recent investigations of B. Baade [18], who found that condensations moving with a velocity of ≈0.1 c appear in the Crab Nebula and that the emission from these regions is due to relativistic electrons. Thus, he observed directly the ejection of matter and relativistic electrons from the atmosphere of the former supernova 900 years after the explosion.

However, this does not lower the importance of investigating the polarization of the radiation from unstable stars since this would give valuable information on the structure of the magnetic field in the regions of nonthermal emission, as well as on the energy spectrum of the relativistic electrons. The results of such measurements must be treated with caution,since it is possible that the nonthermal emission is localized in a large number of "grains," as it appears to be the case in the solar atmosphere [19]. In this case the total radiation may be unpolarized. The following is a good example of this. After I. S. Shklovskii suggested that the emission from the jet in M 87 [20] is produced by relativistic electrons and showed the necessity for polarization measurements, the first measurements gave a negative result because the combined radiation from all three condensations was analyzed, while in fact the directions of polarization are different for each region. Only after Baade's investigations [21] was it established that the polarization is about 30%.

The investigation of the polarization in the various regions of the continuous spectrum, as well as in absorption and emission lines, can be of great interest. In the latter investigations, as was shown by the author, one can expect not only the presence of the Zeeman effect, but also a polarization of another type, which under certain conditions can represent the polarization of the nonthermal radiation component. (For details, see [22]). Let us note that in the recently published papers of a number of English and American astrophysicists – Baade, G. Burbidge, E. Burbidge, Hoyle, Fowler, and Christy [23, 24] – it is also suggested that the radiation of supernovae is nonthermal in nature. However, the arguments presented in favor of this correct conclusion appear to be inconclusive (for details, see [22]). In [24] the above authors come to the conclusion that the radiation of supernovae is radiation from relativistic electrons – a conclusion which was obtained by us in 1953 [2].

The hypothesis of "light-emitting" electrons as applied to chromospheric flares [1] has been confirmed through a very interesting investigation of A. B. Severnyi, who discovered the appearance of "grains" of continuous emission in the chromosphere and photosphere of the sun [19]. A preliminary analysis of this phenomenon leads to the conclusion that the continuous emission has a nonthermal origin and, apparently, is produced by relativistic electrons.

In a series of papers the author has suggested that nonthermal emission also plays an important role in the case of novae [4, 5].

In [25] it was shown that the rapid variations in the luminosity of N. Lac 1950 are undoubtedly of a nonthermal nature. The aim of the present paper, which is the sequel to [25], is twofold:

1. To show that a number of observational data on novae provide direct evidence for the presence of a nonthermal component in the radiation of novae.

2. To investigate the influence of the nonthermal radiation on the important features of unstable-star spectra and to compare the results obtained with observation.

1. Nonthermal Radiation and Rapid Luminosity Variations of Novae

Photographic [26] and photoelectric [27,28] observations of the variation of the luminosity of N. Lac 1950 showed the presence of rapid changes in the luminosity, during which the brightness changed by $0^m.2$ over a period of 20 min. Instances of rapid random oscillations of the luminosity with an amplitude of $0^m.2$-$0^m.3$ were also reported in [27]. The probable error of these measurements was $0^m.03$. Evidence of even faster oscillations, during which the luminosity changed by $0^m.96$ in 16 minutes, is presented in [29]. On the basis of the known absolute magnitude and temperature of the star during these variations of the luminosity, we can determine the required rate of change of the photosphere radius if we assume that the radiation is of thermal origin. We must also take into account the well-known fact that the luminosity increase is accompanied by a decrease of the temperature of the novae. The necessary velocity is found to be about 10^9 cm/sec, which is considerably greater than the velocities of the absorption systems.

The following calculation shows that the thermal mechanism is totally inadequate to account for the rapid variations of the luminosity of N. Lac 1950.

The following variations of luminosity were observed during the Julian day 24 33 316: $316^d.16$, m = $7^m.50$ and $316^d.17$, m = $6^m.54$.

The distance modulus according to [30] is $12^m.5$, so that at these times the absolute magnitudes are -5^m and $-5^m.96$, respectively.

In order to evaluate the brightness in the continuum it is necessary to correct the values of the brightness given for the influence of the emission lines. This can be done with the help of a table published by B. A. Vorontsov-Vel'iaminov [31].

Using the fact that the absolute magnitude at the maximum was $-7^m.5$ and that the decrease of luminosity at the times considered is $2^m.5$ and $1^m.54$, respectively, we find that the correction for the effect of the emission lines is $0^m.3$ and $0^m.2$ (both positive). Thus, at these times the absolute magnitudes, corrected for the emission in the lines, are equal to $-4^m.97$ and $-5^m.94$.

It is necessary to know the temperature of the star to determine the radius of the photosphere. It is known that after the maximum there are oscillations of the temperature of novae during the secondary luminosity oscillations; the effective temperature decreases with increasing luminosity. Larssen-Leander [32] has determined the temperature of N. Lac 1950 during the secondary oscillations of the luminosity and his results are presented in Table 29. Using these results, we find that $T_1 = 2.2 \cdot 10^4$ deg.

We can determine the value of T_2 from the data of [32], with the help of the theory of extended envelopes, as the mean of the values obtained by Zanstra's method: $T_2 = 1.7 \cdot 10^4$ deg.

We use the well-known equation to determine the radius:

$$\log R_p = \frac{7340}{T} - 0.23 - 0.2 M_{pg} + 0.5\log(1 - 10^{14700/T}). \qquad (1)$$

Substituting the numerical data into Eq. (1), we get $\log R_p^{(1)} = 0.995$, $R_p^{(1)} \approx 10\, R_\odot$; $\log R_p^{(2)} = 1.3$, $R_p^{(2)} \approx 20\, R_\odot$. From these results it is easy to find the rate of change of the photosphere radius. This is found to be $\Delta R_p/\Delta t \approx 8 \cdot 10^8$ cm/sec. This value is considerably greater than the velocity of all the absorption systems of the nova. Therefore, we come to the conclusion that the emission responsible for the rapid luminosity variations has a nonthermal nature. It must be noted that the rate of luminosity variations, described above, is comparable to that of the luminosity variations of the UV Cet stars; the latter variations are undoubtedly of nonthermal origin [3, 4]. In connection with this conclusion, based on completely reliable observational data, it is of interest to consider the earlier observational data which lead to the conclusion that similar rapid luminosity variations were also present in the case of N. Her 1934. In analyzing the data on the brightness of N. Her 1934, Wilson [33] noticed the exceptionally large variations in the estimates of the luminosity obtained by different observers at approximately the same time, as well as by one and the same observer.

Frequently, the estimates of brightness obtained by the same observer during the course of a night differed by as much as 1^m. We have sufficient reasons for assuming that these discrepancies were due to rapid random luminosity variations of the same type as were observed in the case of N. Lac 1950. The recent discovery that N. Her 1934 [34] is a binary system and that rapid luminosity variations persist [35], confirms the surmise that in the radiation of former novae there is an intense nonthermal component many years after the explosion. By means of very accurate three-color photoelectric measurements of brightness, which led to the discovery that N. Her is an eclipsing variable of the Algol type, Walker discovered rapid luminosity oscillations with a period of 1 to 40 min and an amplitude of up to $0^m.15$. Periodic oscillations with a period of 1.180 min and amplitude $0^m.07$ were superposed on the above luminosity variations. It appears that the extreme rapidity of these oscillations excludes the possibility of interpreting them as the result of the heating of the photosphere by shock waves moving from the deeper layers of the star. In this case, the time in which the luminosity increases, and especially during which it decreases, would be considerably longer.

Of exceptional interest is the fact that these periodic luminosity variations occur during a definite part of the period, i.e., as Walker notes correctly, they are associated with a definite region of the stellar surface. The above data provide a direct confirmation of the hypothesis, proposed earlier by the author, that the activity in novae is localized in definite regions.

In connection with this, it must be noted that rapid luminosity variations have also been discovered in the case of MacRae's star +43 deg. [36], whose variability was discovered by P. P. Parenago [37]. Over a period of only 5 min, the change in brightness in the ultraviolet region was as high as $0^m.4$.

J. Greenstein [38] has investigated the spectrum of the star and found that, apparently, it is either a former nova or it is in a prenova stage. It is certain that the rapid luminosity variations of this star are of nonthermal origin.* Thus, we can conclude that even many years after the outburst of a nova, and possibly also before the explosion, energetic nonstationary processes take place in the atmosphere of the star accompanied by the production of relativistic particles whose radiation in the magnetic field of the star results in the observed rapid variations of the luminosity. A strong argument in favor of this conclusion is the fact that the presence of the theoretically predicted activity of this type in the remnants of a supernova [2] was established by Baade, who observed the motion of relativistic-electron condensations ejected from the central star of the Crab Nebula [18].

Recently, the results of Herzog's observations, made in 1935 at Pulkovo and Babelsberg, have been published [39]; from an analysis of the luminosity of N. Her, Herzog concluded that at that time the star was also an eclipsing binary. If this conclusion is confirmed, it will provide a direct proof that the emission in nova explosions is of a nonthermal nature. Indeed, according to [35] the radius of the orbit is $2.1 \cdot 10^{10}$ cm. Further, if the emission is thermal and the increase in luminosity is governed by the expansion of the photosphere, then the radius of the photosphere in 1935, before the sudden fall in luminosity in April, 1935, would have been ten times greater than the radius of the binary star orbit. Thus, both stars would have been inside an opaque expanding envelope and it would have been impossible to observe that they formed a binary system.

The nonthermal nature of emission from novae leads to another interpretation of their light curve. Walker points out that, in view of the difficulty encountered because of the low value for the nova mass obtained from photometric data, it is necessary to consider the possibility that the luminosity oscillations are due to the rotation of a single star with a dark spot on its surface. He excludes this possibility since the large size of the spot disagrees with the short duration of the minimum.

However, if we take into account that the synchrotron radiation of electrons has a nonthermal nature and the luminosity of small regions on the surface of the star can be very high, then the luminosity oscillations can be explained by the rotation of a single star about its axis.

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*Note added in proof. This conclusion is confirmed by the recent discovery of the polarization of the rapid luminosity variations in the ultraviolet spectrum of NX Tau [51] (see footnote on page 720).

2. The Influence of Nonthermal Emission on the Appearance of the Absorption Lines

The presence of intense nonthermal radiation as a component of the continuous emission from novae must manifest itself in their spectra. The nonthermal emission must be localized either in the photosphere or above it.

From the nature of the stellar photosphere it follows that it represents a region in which τ is of the order of unity. Naturally, if the nonthermal radiation were produced below the mass of photospheric layers, it would be converted into thermal radiation as the result of successive absorption and re-emission of the quanta. Of course, we cannot exclude the possibility that when intense nonthermal radiation is produced in a layer which was originally optically dense, the layer becomes transparent. However, in this case the effective height of the photosphere decreases and the nonthermal emission would once again be found above the photosphere.

First V. A. Ambartsumian, in the case of T Tau stars [3], and then the author, in the case of novae [5,40], have investigated the phenomenon of "flooding" of the stellar absorption spectrum or some part of it, which is due to the continuous emission of nonthermal origin.

In this section we will investigate further the consequences of the presence of nonthermal radiation in the reversing layer. If the radiating electrons are localized at the bottom of the photosphere, then in this case both weak and strong absorption lines are observed. If the source of the emission rises into the upper layers of the atmosphere, a weakening of the lines occurs and the residual intensities increase. When the emission source is above the reversing layer (for example, because of the diffusion of the radiating electrons upward), the "flooding" of the spectrum by the nonthermal emission and a complete disappearance of the absorption lines can occur. Just these features are observed in the spectra of a number of novae, particularly those of N. Cyg 1920 [5, 25, 40, 43]. Only the strongest lines are present in a number of absorption systems of this nova. The weak lines of all elements are entirely absent. The main spectrum which appeared on August 22 disappeared on August 23 (at the luminosity maximum) and reappeared on the next day. This phenomenon cannot be accounted for by an increase of ionization,since during this time the temperature of the star was falling. A natural explanation of the presence in the spectrum of only the strong lines can be the existence of nonthermal emission, which "floods" the weak lines, in the lower part of the reversing layer. The high residual intensity of the strong lines can also be explained by nonthermal emission occurring in the reversing layer. A temporary increase in the height of the emission level could have led to the disappearance of a given absorption system and its subsequent reappearance when the height of the emission level once more decreased. The appearance and the temporary disappearance of various absorption systems were also observed in the spectrum of N. Her 1934, where this phenomenon was investigated in detail by D. McLaughlin [41] who emphasized their identity before and after disappearance. He wrote: "The components mentioned simply become invisible and a continuum (or possibly overlying emission?) appears in their place." *

The following remark made by McLaughlin is of great interest: "If by some means we were able to remove the lower atmosphere and leave only the upper layers lying between the observer and the photosphere, we would observe a spectrum similar to the absorption spectrum II of March 1, 1935" [42]. *

In this case, apparently, the explanation of the situation observed is that the source of nonthermal emission rises upward and the nonthermal radiation overwhelms that from the lower layers of the atmosphere.

A rapid change in the sharpness of absorption lines was also observed in the spectra of N. Cyg 1920 [43] and N. Gem 1912 [44]. Thus, the spectrum of N. Gem obtained on March 22.8 was found to consist of only sharp lines, while on March 23.7 the spectrum had a fairly diffuse nature, after which the sharpness once again increased. These important data indicate that the diffuseness of the spectrum cannot be due to turbulence in the envelope. The temporary disappearance of all absorption spectra during the secondary minima was observed in the case of N. Aql 1918 [45]. Since all of the lines were affected, including those corresponding to a high degree of ionization, the reason for the disappearance, apparently, cannot be an increase in ionization associated with the increase of temperature during the minima. Moreover, in the intermediate stages, between the maxima and the minima of luminosity, there is an absence of intermediate spectra which would have indicated a gradual increase in the ionization and,associated with this, a weakening of the lines. The same situation was also observed in the case of N. Per 1901 [46].

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* Translated from Russian — quotation not verified — Publisher.

It appears that in all of these cases the absorption spectra were "flooded" by the continuous emission of nonthermal origin. After the radiating electrons, on reaching the outer atmospheric layers of the nova, have lost their energy or have diffused into more rarefied regions of the envelope, the absorption system becomes visible again.

In connection with the model considered above, the following observation of McLaughlin, referring to the disappearance and reappearance of absorption systems in the spectra of N. Her 1934, is of great interest. He points out (this is clearly shown in the figure given in the paper cited) that when the absorption systems reappeared, their intensities were found to be considerably greater than before the disappearance, as though their intensities increased monotonically during the period in which they were invisible [41]. Neither the expanding-envelope model, nor the hypothesis of continuous ejection can provide an explanation of this important fact.

The decrease in the energy of the radiating electrons in a region of the upper photosphere where a given absorption system is formed, so that the latter is no longer "screened" from the observer, and the strengthening of the nonthermal emission from the layers beneath the reversing layer can provide a natural explanation of the observed situation. It is possible that there are additional factors present, particularly the transformation of absorption lines into emission ones. This question will be discussed in the next section.

In connection with the presence of a nonthermal component in the radiation of novae, it is of interest to note a further feature of their spectra. In one of the absorption systems of N. Cyg 1920 – spectrum I – the absorption lines NII, OI, and others had a different appearance in different regions of the spectrum [43]. Clearly marked in the visible region of the spectrum, they were absent in the photographic region. The TiII lines of equal laboratory intensity were considerably stronger in the visible region of the spectrum. It is interesting that other absorption systems, observed at the same time in the spectrum of the star, did not exhibit this behavior. Wright [44] has noted a similar phenomenon in two absorption systems a and b in the spectrum of N. Gem 1912. It is reasonable to assume that the different appearance of the lines in the different regions of the spectrum, observed in some absorption systems, can be accounted for by a different spectral composition and a different distribution with height of the nonthermal radiation in those surface zones of the star with which these absorption systems are associated. From this point of view, it is of great interest to consider Walker's results [27] indicating that the rapid luminosity variations are sometimes accompanied by equally rapid variations in the color of the star, during which the color index of a nova changed by $0^m.43$ in the course of only half an hour. Undoubtedly, these rapid variations of color were associated with the rapid variations in the spectral composition of the nonthermal radiation.

In connection with the presence of rapid variations in the color of N. Lac 1950, it is interesting to note the earlier investigations of Oheler [47], who discovered that the spectrophotometric temperature of N. Her 1934 varied by 3400° in less than an hour. At the time these data were treated with suspicion, but now we are led to believe that they were associated with variations in the color of the star, similar to those discovered by Walker in the case of N. Lac 1950.

In conclusion, we must discuss the extremely interesting investigations, carried out by Abt [48] of the spectrum of a star of the type RV Tau – U Mon.

During variations of luminosity, whose amplitude sometimes exceeds 2^m, color variations are found to be small and, further, uncorrelated with the luminosity variations. On the other hand, the luminosity oscillations are opposite in phase to the variations in the atmosphere radius. The greatest expansion corresponds to the minima, while the greatest contraction corresponds to the maxima. There are no grounds for expecting an appreciable phase difference between the displacements of the photosphere and the reversing layer of the star, since the dilution effect is not present in its spectrum.

Thus, the explanation of the luminosity oscillations in the case of U Mon in terms of a thermal mechanism encounters great difficulties. On the other hand, from spectroscopic observations Abt found that the abundance of iron atoms changes by a factor of more than 1000, while the degree of ionization remains almost constant. Thus, it is necessary to assume that both T_e and P_e (the latter varies by a factor of 1000 !) always vary in such a way that the degree of ionization remains constant. Such an assumption is very artificial and unrealistic. H. Abt points out that the variations in the atmospheric density cannot account for the observed changes in the abundance, since the abundance decreases with decreasing radius of the atmosphere. The assumption that the abundance of iron changes as the result of FeII ionization with increasing excess of ultraviolet radiation with λ in the region of 767 A is also in disagreement with observations. This effect is selective in character, while the observed weakening of the lines extends over all the absorption lines in the spectrum of the star.

Abt considers that the variation of the coefficient of continuous absorption in the atmosphere of the star is a probable cause of the variation in Fe abundance. This explanation, however, appears artificial, since it is hard to imagine that such a considerable change in the continuous absorption could occur without any changes in the degree of ionization of the atmosphere. Moreover, this explanation is unconnected with the luminosity oscillations which, apparently, have a nonthermal origin. The rising of the regions responsible for the nonthermal emission into the upper layers of the atmosphere, as occurs in T Tau and UV Cet stars, can account for this remarkable situation. It appears that in this case there is an analogous "flooding" of the lines by the continuum of nonthermal origin, but the "flooding" does not lead to a complete disappearance of the lines. Earlier, from a consideration of the various features observed in the spectra of RV Tau stars, we proposed that an intense nonthermal component, which governs the luminosity oscillations [4], is present in the radiation of these stars. We now see that with the help of this hypothesis we can explain also the essential features of their spectra.

3. The "Coefficient of Nonthermal Excess" and the Main Properties of the Spectra of Unstable Stars

In addition to its direct influence on the appearance of the absorption lines in the spectra of unstable stars, which we have discussed above, the nonthermal radiation can also significantly affect the stellar spectra in other respects [25, 22].

It is obvious that sufficiently intense nonthermal radiation, characterized by a definite spectral composition, can have a significant influence on the distribution of atoms in excited states. Moreover, induced transitions become important for atoms situated in the field of intense nonthermal radiation; as a result of this, the width of the lines can increase considerably. It seems that in the theory of stellar spectra considerable importance has to be attached to the fact that the line width becomes a function of the radiation flux, i.e., the width depends on the level in the atmosphere where the absorbing or emitting atoms are located and, in the case when the nonthermal emission is localized in definite regions, on the coordinates of a region on the surface of the star.

Further, under certain conditions the nonthermal radiation can manifest itself not only through the polarization of the continuous spectrum, but also through the polarization of the light in absorption and emission lines.

The probability of a transition from a level n to a lower level m is given by the well-known expression

$$W_{nm} = A_{nm} + B_{nm}u_{\nu}, \tag{2}$$

where A_{nm} is the probability of a spontaneous transition, B_{nm} that of an induced transition, and u_{ν} is the radiation density at the appropriate frequency.

In the case when the radiation is described by Planck's Equation, Eq. (2) can be rewritten in the following form:

$$W_{nm} = A_{nm}\left(1 + \frac{1}{e^{h\nu/kT} - 1}\right). \tag{3}$$

The second term of the expression inside the brackets corresponds to the induced transitions and becomes important only in the region where the Rayleigh-Jeans Law is applicable. In the case when the atoms are in the field of intense nonthermal radiation, the equation must be rewritten as follows:

$$W_{nm} = A_{nm}\left(1 + \frac{\varepsilon_{nm}}{e^{h\nu/kT} - 1}\right), \tag{4}$$

where ε_{nm} is the ratio of the spectral density of the radiation to that given by Planck's Law for the same frequency. Here, we assume that near the frequency corresponding to the transition m → n, the energy distribution can be approximately described by the Planck function, i.e., characterized by a color temperature. Following

[25, 22], we will call this quantity the "coefficient of nonthermal excess." For astrophysical purposes the expression given is more convenient than the usual one which contains the radiation density, since frequently the color temperature of the star is approximately known from the spectral class of the star or from the color index, or from the more accurate spectrophotometric data. Formally, the quantity ϵ_{nm} can be compared with the dilution coefficient and,in virtue of this, it may be called the "antidilution coefficient." However, there is one important difference which arises because the dilution coefficient is mainly determined by geometrical factors and is only slightly associated with the deviation of the spectral composition of the radiation emitted by the source from that of black-body radiation. The "coefficient of nonthermal excess," on the other hand, is primarily a function of the spectral composition of the nonthermal radiation which, in addition, can vary with the time or with location on the star, as well as with height in the atmosphere of the star. Therefore, the introduction of the coefficient of nonthermal excess may necessitate a more drastic revision of the theory of stellar spectra then the revision required for the theory of radiation from diffuse matter if the dilution coefficient is taken into account.

Thus, we can state that the introduction of the "coefficient of nonthermal excess" into stellar-atmospheric physics provides us, in a sense, with an extra degree of freedom which determines the main properties of the absorption and emission spectra of unstable stars.

If $\varepsilon_{nm}/e^{\eta\nu|kT}-1\gg 1$, then the dominant role is played by the induced transitions which will occur more frequently than the spontaneous transitions. The expression giving the damping constant in the presence of nonthermal radiation is also of great interest:

$$\gamma_m = \sum_n A_{nm} + \sum_n B_{nm}\bar{u}(\nu_{nm}) + \sum_{n'} B'_{nm}\bar{u}(\nu_{n'm}). \tag{5}$$

The first term corresponds to the spontaneous emission, the second to induced emission, and the third to absorption. If we assume that in a definite spectral region the intensity distribution can be described by a color temperature and the nonthermal radiation can be characterized by a "coefficient of nonthermal excess" ϵ_{nm}, the expression for the damping will take the form

$$\gamma_m = \sum_n A_{nm}\left(1 + \frac{\epsilon_{nm}}{e^{h\nu/kT}-1}\right) + \sum_{n'} A_{mn'}\frac{g_{n'}}{g_m}\frac{\epsilon_{mn'}}{e^{h\nu/kT}-1}\,. \tag{6}$$

The first term corresponds to emission processes and the second to absorption processes. As is known, the line width depends not only on the probability of a transition between two appropriate levels, but also on the damping constant of each of these levels, which depends on all the possible transitions from these levels to other levels. The presence of nonthermal radiation, which is characterized by different values of the "coefficient of nonthermal excess" for different transitions, can lead to a considerably stronger dependence of the line width on the probability of transitions from the initial and final levels. A change in the intensity and spectral composition of the nonthermal radiation can lead to rapid variations in the line width even in the case when these oscillations are limited to a region of the spectrum far from the position of the line and, in general, outside the visible region of the spectrum.

The following simple mechanism, associated with the presence of nonthermal radiation, may lead to an increase in the number of atoms in the excited states and the appearance of emission lines. The excess of nonthermal radiation in the ultraviolet region of the spectrum can lead to additional ionization of certain atoms. The energy of the photoelectrons produced is quickly shared with the thermal electrons and,as a result of this, Maxwell's distribution of velocities is established. The excess of nonthermal radiation in the infrared region of the spectrum leads to an increase in the number of induced recombinations from the excited levels, as a result of which the appearance of emission lines is possible. Thus, an increase in the radiation density can lead to an increase in the number of atoms in the excited states and a conversion of absorption into emission lines.

Before we proceed to a discussion of the observational data on the spectra of nonstationary stars in connection with the presence of an excess of nonthermal radiation, it is necessary to estimate quantitatively the value of the "coefficient of nonthermal excess" for novae and to evaluate the role played by induced transitions.

Photographic data on the luminosity variations of N. Lac 1950 show that the luminosity changed by $0^m.96$ in 16 min. As was stated above, this rapid variation of luminosity could have been due primarily to nonthermal radiation. Let us assume that the nonthermal radiation is no longer present when the luminosity falls to the secondary minimum. This will give us the lower limit to the "coefficient of nonthermal excess." It follows that at the maximum of luminosity the nonthermal component was 1.5 times as intense as the thermal radiation. Assuming that the area of the nonthermal emission zone is 10% of the total area of the star, we find that the average value of the "coefficient of nonthermal excess" for the optical region was $\bar{\epsilon} = 15$.

In order to determine the role played by induced transitions in the case of various spectral lines, we will give a table of the values of $1/e^{h\nu/kT} - 1$ for various series in the hydrogen spectrum for two values of the temperature: $T_1 = 8 \cdot 10^3$ deg and $T_2 = 3 \cdot 10^4$ deg. The first value corresponds approximately to the luminosity maximum, the second to a considerable decrease in luminosity after the maximum, when the temperature has already increased.

TABLE

The Values of the Quantity $1/e^{h\nu/kT} - 1$ for $T_1 = 8 \cdot 10^3$ deg and $T_2 = 3 \cdot 10^4$ deg

Initial state / Final state	$n = 1$		$n = 2$		$n = 3$		$n = 4$		$n = 5$	
	T_1	T_2	T_1	T_2	T_1	T_2	T_1	T_2	T_1	T_2
2	$2.8 \cdot 10^{-7}$	0.018	—	—	—	—	—	—	—	—
3	$7.2 \cdot 10^{-8}$	0.008	0.07	0.89	—	—	—	—	—	—
4	$8 \cdot 10^{-9}$	0.007	0.05	0.59	0.68	3.45	—	—	—	—
5	$5 \cdot 10^{-9}$	0.006	0.04	0.49	0.32	2.17	1.8	8.4	—	—
6	$4 \cdot 10^{-9}$	0.005	0.03	0.45	0.23	1.8	1.0	5	3.7	16.7

The value of this quantity [see Eq. (1)] gives the ratio of the number of induced transitions to the number of spontaneous transitions. If at a frequency corresponding to a given transition, excess nonthermal radiation characterized by the value of the coefficient ϵ_{nm} is present, then this quantity must be multiplied by ϵ_{nm}. In the case of N. Lac 1950, since the change in the luminosity by $0^m.96$ occurred when the luminosity of the star had decreased by more than 2^m, we will use the values of $1/e^{h\nu/kT} - 1$ for the temperature $T = 3 \cdot 10^4$ deg. It may be seen from the table that for $\epsilon = 15$, spontaneous transitions continue to be predominant in the Lyman series, while in the case of the Balmer and higher series, induced transitions assume the dominant role.

Thus, we can assume that the emission lines in the Balmer and higher hydrogen series, as well as the lines of other elements in the visible and infrared regions of the spectra of novae, sometimes actually have the nature of negative absorption lines. As is known, negative absorption lines have a number of very interesting features. Their intensity increases exponentially with height in the atmosphere until the total number of transitions from the particular level becomes equal to the number of transitions to this level. Then, the further increase in the intensity is proportional to the thickness of the layer. Since the quantity $1/e^{h\nu/kT} - 1 \gg 1$, for transitions from the continuum near the corresponding series limit to all levels starting from the third, we can consider that the number of induced recombinations is approximately equal to the "coefficient of nonthermal excess." We therefore see that the nonthermal radiation can affect very considerably the number of atoms in the excited states and, consequently, affect the relation between emission and absorption, the intensity of the emission lines, as well as the value of the Balmer decrement. In particular, we can expect the transformation of absorption lines into emission lines in the spectra of those unstable stars in which we suspect the presence of an intense and variable continuous emission of nonthermal origin.

Let us now compare the results of our model with the data of the observations of nonstationary stars. The characteristic feature of the spectrum of N. Cyg 1920 was the remarkable anomaly of the Balmer decrement in the case of the various emission spectrum systems [43]. From the width of the emission lines it was established that system VI was represented only by the H_α line, system IV by H_α and $H\beta$, system III by H_α, $H\beta$, and H_γ, and system II by lines up to H_δ. Such a steep Balmer decrement has little resemblance to the Balmer decrement for nebulae. As is known, the Balmer decrement in the case of gaseous nebulae depends little on the electron temperature. Self-reversal operates in the opposite direction, while the assumption that the anomalous steepness of the Balmer decrement is the result of light absorption has to be rejected both because the Balmer decrement varies with time and because it has different values for different systems. The Balmer decrement of N. Lac 1950 was also strongly correlated with the luminosity oscillations [32].

Evidently, the reason for the anomalous values of the Balmer decrement is the anomalous distribution of atoms among the excited states. The excess nonthermal radiation and the associated induced transitions to excited levels can be the source of the anomaly. The intensity of a line also depends on the nonthermal radiation with the same frequency as the line, i.e., an increase of the intensity at the line frequency leads to an increase in the number of atoms in the upper level.

During the secondary luminosity oscillations of N. Aql 1918 [45] there was a considerable weakening of the absorption lines and a relative strengthening of the emission lines in its spectrum. During a number of the minima the absorption lines disappeared completely. In these cases, in addition to an increase of the emission level, there was also, apparently, an increase in the number of atoms in excited states and a decrease of the number of atoms in the lower levels.

Of exceptional interest are the observations of McLaughlin [49] on the transformation of absorption lines in the spectrum of N. Her 1934 into emission lines. He noted instances when regions of weak emission appeared in exactly the same positions as those of the Ti II and Fe II lines of spectrum I.

In [49], McLaughlin writes: "During this time, the most remarkable feature of the spectrum was the appearance of regions of weak emission in exactly the same positions as those of the absorption lines Ti II and Fe II of spectrum I. This gives us grounds for assuming that the strengthening of the violet component of the allowed FeII emission lines takes place as the result of the replacement of absorption by emission. The forbidden lines do not appear in absorption and, therefore, their violet component is considerably stronger at an earlier time as, for example,[O II]." McLaughlin proposes that the appearance of emission in place of absorption lines also occurred in the case of the disappearance of some absorption systems of N. Her [41].

Let us note that in the spectra of other unstable stars whose radiation has a nonthermal component — stars of the type T Tau, R CrB, and U Gem, a similar transformation of absorption into emission lines also occurs. The specific spectral features of T Tau stars — an anomalous brightness in the ultraviolet region of the spectrum and also the fact that the images in the violet light of the T Tau stars, which have an anomalously high intensity in the ultraviolet region of the spectrum, are weaker than the images of white dwarfs which have the same yellow and ultraviolet regions of the spectrum [50] — this all points to the presence of nonthermal radiation with a dependence of the intensity on wavelength different from that of the thermal radiation. The fact that strong emission in the ultraviolet only occurs in the case of stars with a strong emission in H_α becomes very important. It should also be noted that there is a strong correlation between the presence of an ultraviolet excess and the value of the Balmer decrement which is always small for stars with an intense ultraviolet emission. Evidence for a very unusual distribution of atoms among the excited states is provided by the observation that in the spectra of AU and BC Ori stars only emission lines occur, without any traces of absorption, even if the energy distribution for $\lambda > 3800$ A roughly suggests the distribution in the spectra of the K and early M dwarfs.

The observational data on the T Tau stars also indicate that there is a strong connection between the spectral composition and intensity of the nonthermal composition, on the one hand, and the intensity of the emission lines, on the other.

Investigations of the spectra of N. Her 1934 [42] and N. Per 1901 [46] have shown that,together with the oscillations in the luminosity, there are oscillations in the intensity of the allowed and forbidden lines. This phenomenon cannot occur as the result of variations in the amount of matter ejected, since in this case there would be a considerable difference in phase between the oscillations in the intensity of the allowed and forbidden lines. Inasmuch as a shift is not observed, the oscillations in the intensity of the emission lines are apparently due to a change in the population density of the excited levels and this change, in turn, is caused by the induced transitions taking place under the action of intense nonthermal radiation. It is necessary to make one remark about the spectral composition of the nonthermal radiation. Apparently, it would be an oversimplification to calculate the nonthermal radiation spectrum on the assumption that the only operative factor is the intensity distribution of the synchrotron radiation emitted by the relativistic electrons. In their paper, already referred to above, Haro and Herbig [50] point out the existence of a broad maximum in the region of 3700 A. It seems that in the region of shorter wavelengths, the spectrum is terminated more abruptly than it would have been in the case of pure synchrotron radiation. Possibly, in order to calculate more accurately the spectral composition of the total radiation from a star, we should take into consideration that the nonthermal radiation must penetrate a certain thickness of the reversing layer and, in those regions of the spectrum where the absorption coefficient has a maximum, the radiation will be partially converted into thermal radiation (at these wavelengths the photosphere effectively extends higher; the radiation with these frequencies is partially reflected inwards and is there converted into radiation of other frequencies). The smaller the value of the "coefficient of nonthermal excess," the greater will be the change in the spectral composition of the radiation produced by these factors. The final remark pertaining to this section can be stated as follows: the exponential increase of the radiation flux in the negative absorption lines with height, until the number of atoms entering a given level is equal to the number leaving it, and the linear increase of the flux with further increase of the thickness, can lead to the situation in which most of the radiation in an emission line can arise in a limited region, which is sometimes small, and not throughout the whole extent of the atmosphere. The radiation from the upper regions of the atmosphere is limited because of the decrease of density with height.

The magnitude of the damping, γ_m, and the number of atoms in the excited states must vary considerably with height; because of this, the observed line profile will be the superposition of line profiles corresponding to the different levels in the atmosphere. If, for example, the density of nonthermal radiation increases in the infrared region, i.e., the spectral region governing the rate of induced electron transitions from a free to the bound state which is the initial state for the emission of a given line, then the number of transitions to these states will increase and, as a result of this, the effective level at which the emission line is produced will occur higher in the atmosphere. If the density decreases, a decrease in the height of the level is observed. If there is an increase in the intensity of the nonthermal radiation at the frequency of the line itself, then the number of transitions to the level becomes equal to the number of transitions from it in a lower part of the atmosphere. Thus, variations in the composition and intensity of the nonthermal radiation produce a change in the height of the effective level at which emission lines are produced. If there is a velocity gradient in the atmosphere, we will observe radial-velocity oscillations. These will also be observed if there is an appreciable gradient of the magnetic field intensity in the atmosphere. Changes in the effective line-production level, associated with oscillations in the nonthermal radiation, will in this case result in apparent oscillations of the magnetic field intensity. This effect is possibly observed in the spectra of some unstable stars, in the case of which periodic variations have been observed of radial velocity, magnetic field, line intensity, etc. This effect can also lead to changes in the degree of ionization, i.e., to changes of spectral class, since the degree of ionization must vary with height in the atmosphere.

Finally, it is necessary to point out that if the excited states, transitions from which give rise to emission lines, as well as the lower level associated with the production of absorption lines, are occupied as the result of induced transitions brought about by polarized nonthermal radiation, then the corresponding lines may also exhibit a certain degree of polarization under favorable conditions [22]. Investigations of this type in the case of novae and supernovae, as well as of other types of unstable stars, appear to be very desirable.

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ON THE EVOLUTIONARY INTERPRETATION OF THE MAIN SEQUENCE OF STARS

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In this paper are presented the main arguments which lead to the idea of corpuscular emission from the early stars of the main sequence; the rate at which the stellar masses decrease is estimated and it is shown that the process considered can only be characteristic of the stars in the first part of the main sequence. From the observed luminosity function for these stars their initial mean statistical luminosity function is derived which, in contrast to S. A. Kaplan's earlier results, is found to have two maxima, the main maximum occurring at the bolometric magnitude $M = 0$ and the secondary maximum at $M = -4.5$. The intermediate minimum, at $M = -3$, is in agreement with A. I. Lebedinskii's theoretical result on the upper limit to the initial stellar masses $(6\mathfrak{M}_\odot)$. The luminosity functions for the stellar associations in Cygnus and Orion are close to the derived initial luminosity function. The calculation of the ages of the stars of various spectral classes belonging to the first part of the main sequence has led to a quantitative explanation of the dispersion of their peculiar velocities in terms of a gravitational perturbation produced by the more massive galactic inhomogeneities. In connection with the evolutionary process investigated, the lower limit to the average helium abundance is estimated; this estimate is found to agree with the observed average abundance of helium in B-star envelopes. If the stars of the second part of the main sequence evolve without a loss of mass and without mixing, while the red giants represent the subsequent stage in the development of the heavier of these stars, then the average age of both is consistent with the dispersion of their peculiar velocities in the galactic plane.

1. If stars evolve appreciably during their lifetime τ, as it is now in fact universally accepted, and in general move continuously across the Hertzsprung-Russell diagram (with the possible exception of brief jumps), then the very existence of all the sequences on this diagram, among them the main sequence, must have some bearing on stellar evolution.

However, a special analysis is necessary to discover the nature (direction and rate) of stellar evolution and to investigate the initial conditions prevailing during the formation of a star, and the terminal stages (the arrival of a star at the main sequence and the possible departure from it); in order to obtain an unambiguous conclusion from this analysis, it is totally inadequate to base the calculations, as it is sometimes done [1], on the use of only the luminosity function $\varphi(M)$, which is the relative number of stars of the type investigated with absolute bolometric magnitudes lying between M and M + dM:

$$dN/N = \varphi(M)\,dM, \qquad (1)$$

where N is the total number of stars being investigated (in the Galaxy or per cubic parsec). The observed luminosity function provides one possibility of testing and selecting evolutionary hypotheses, but only when these hypotheses determine both the direction and the rate of evolution (if only in relative units of time), i.e., when, starting from some assumptions about the initial and final conditions, it is possible to calculate the behavior of the theoretical luminosity function and to compare it with observations.

The Vogt-Russell theorem [3] is very important for the determination of the evolutionary paths of main-sequence stars with thermonuclear sources of energy; according to this theorem, the structure of the star is unambiguously governed by its mass $\mathfrak{M}$ and chemical composition if the energy production depends only on the local values of the density ρ, the temperature T, and the chemical composition. The concentration of stars along the linear main sequence in the Hertzsprung-Russell diagram, as well as the existence in the case of these stars of almost functional empirical mass-luminosity and mass-radius relationships [4], indicates that the masses of the stars considered are a function of their chemical composition which, in general, can be described, as is well known [3], by the values of the relative abundances of hydrogen (X), helium (Y), and heavier elements (Z), the latter taken for definiteness in the form of the Russell mixture. In this case the mean molecular weight of the stellar matter is

$$\mu = \frac{1}{2X + 3/4Y + 1/2Z}, \tag{2}$$

where

$$X + Y + Z = 1. \tag{3}$$

Assuming that thermonuclear reactions of the carbon-nitrogen cycle are the source of energy in the main-sequence stars, A. B. Severnyi [5] came to the conclusion that in these stars a decrease in mass leads to a decrease in the hydrogen abundance and an increase in the helium abundance and that the mean molecular weight can be expressed as a function of the mass by the following relation:

$$\mu/\mu_\odot \approx (\mathfrak{M}/\mathfrak{M}_0)^{-0.225}, \tag{4}$$

which holds very well in the range from 30-50 to 0.3-0.1 solar masses. The estimates of the hydrogen and helium abundances made by a number of authors in the case of typical main-sequence stars are in adequate qualitative agreement with this conclusion, which V. G. Fesenkov [6] compares with the fact that interstellar matter, evidently the raw material out of which stars are formed, has a particularly high abundance of hydrogen. This is also in agreement with the suggestion made by B. A. Vorontsov-Vel'iaminov [7] that an appreciable amount of the diffuse matter observed by us is the result of corpuscular emission from massive hot stars (having a low helium content). As a star evolves, the thermonuclear reactions which result in the conversion of hydrogen to helium must lead to an increase in the molecular weight. However, if the star is to remain on the main sequence its mass must decrease in virtue of a relation of the type of Eq. (4). Therefore, V. G. Fesenkov's hypothesis that corpuscular emission is the main factor in stellar evolution is a natural generalization of the empirical data. This is an important conclusion, since the derivation of a relation of the type (4) on the basis of initial conditions, and not from the theory of stellar evolution, does not appear to be possible.

2. For a star with a homogeneous chemical composition, the rate at which the hydrogen content decreases with time is determined from its luminosity L and mass by the obvious equation

$$\frac{dX}{dt} = -\frac{L}{\alpha\mathfrak{M}}, \tag{5}$$

where

$$\alpha = 6.4 \cdot 10^{18} \text{ ergs/g} \tag{6}$$

is the energy liberated by the conversion of 1 g of hydrogen into helium. Taking the mass to be a function of the hydrogen content, we find

$$\frac{d\mathfrak{M}}{dt} = \frac{d\mathfrak{M}}{dX}\cdot\frac{dX}{dt} = -\frac{d\mathfrak{M}}{dX}\,\frac{L}{\alpha\mathfrak{M}}, \tag{7}$$

i.e.,

$$d\mathfrak{M}/dt = -kL, \tag{8}$$

where

$$k = \frac{1}{\alpha} \cdot \frac{d \ln \mathfrak{M}}{dX}. \tag{9}$$

Assuming that the relative concentration of heavy elements remains unchanged in the course of the evolution of the star by corpuscular emission,

$$Z = \text{const}, \tag{10}$$

and taking into account Eqs. (2)-(4), we can easily calculate the coefficient k from Eq. (9):

$$k = \frac{1}{\alpha} \frac{d \ln \mathfrak{M}}{d\mu} \frac{d\mu}{dX} \approx -\frac{1}{0.225\,\alpha} \frac{d \ln \mu}{d\mu} \frac{d}{dX} \frac{1}{{}^5/_4 X + {}^3/_4 - {}^1/_4 Z} = \frac{50}{9\alpha} \mu. \tag{11}$$

The molecular weight of the ionized stellar material can vary within the comparatively narrow limits of ½ (pure hydrogen) and 2 (only heavy elements). Thus, in Eq. (8), where the luminosity L can vary by several orders of magnitude, the coefficient k remains practically constant and is approximately equal to

$$k \approx 10^{-18} \text{ g/erg} \tag{12}$$

($\mu \approx 1$), i.e., the loss of mass as the result of corpuscular emission from the star must exceed by three orders of magnitude the loss of mass corresponding to the energy radiated:

$$\frac{d\mathfrak{M}'}{dt} = -\frac{1}{c^2} L\,(c = 3 \cdot 10^{10} \text{ cm/sec}). \tag{13}$$

The expressions for the coefficient k, derived earlier by A. G. Masevich [9, 10], yield values of k which are of the same order of magnitude as (12) and which vary little in the course of stellar evolution; further, if we use a power law with a variable index n as an approximation to the mass luminosity relation:

$$L/L_\odot = (\mathfrak{M}/\mathfrak{M}_\odot)^n \tag{14}$$

and take into account a change in this index for a transition from the first part of the main sequence to the second [4], then the variation of the coefficient k becomes even less important.

Therefore, without going into a discussion of the possible physical mechanisms for the corpuscular emission, it is reasonable to assume, as was done originally by V. G. Fesenkov [8], that the rate at which the star loses its mass is proportional to its luminosity, i.e.,

$$k = \text{const}. \tag{15}$$

In this case from Eq. (11) and expressions (2) and (3) we get

$$k = -\frac{5}{4\alpha} \mu^2 \frac{d \ln \mathfrak{M}}{d\mu}, \tag{16}$$

i.e., instead of Eq. (4) we obtain the following relation between the molecular weight and the mass of the star:

$$\frac{1}{\mu} - \frac{1}{\mu_\odot} = \frac{5}{4\alpha k} \cdot \ln \frac{\mathfrak{M}}{\mathfrak{M}_\odot}. \tag{17}$$

Substituting into this expression A. B. Severnyi's data [5] for the sun ($\mu_\odot = 0.818$) and for ζ Aquarii ($\mu = 0.590$, $\mathfrak{M} = 8.1\,\mathfrak{M}_\odot$), we find

$$k = 0.86 \cdot 10^{-18} \text{ g/erg}. \tag{18}$$

The existence of an upper and a lower limit to the masses of the stars considered follows from Eq. (17). The molecular weight has its lowest value in the case of ionized hydrogen: 0.5. When the usual admixture of ionized heavy elements is present (Z = 0.12 [11]), this minimum value becomes raised to 0.549. As a result we get the following upper limit to the masses of the stars belonging to the main sequence:

$$\mathfrak{M} < (14-31)\mathfrak{M}_{\odot}. \tag{19}$$

In the case of stars containing only helium or of stars with the same admixture of heavy elements (after all the hydrogen has been exhausted), the molecular weight can be as high as 1.333-1.389, and this gives the corresponding lower limit to the masses:

$$\mathfrak{M} > (0.5-0.6)\mathfrak{M}_{\odot}. \tag{20}$$

The order of magnitude of the limits given by (19) and (20) is reasonable and within these limits Eqs. (4) and (17) give practically the same value for μ (when the errors in the molecular-weight determinations are taken into account), i.e., the calculated mass-luminosity and mass-radius relationships which, following A. B. Severnyi [5], are derived from Eq. (17), and not (4), of the corpuscular-emission theory must be in good agreement with the empirical relationships for stars of the main sequence.

Properly speaking, Eq. (20) implies that stellar evolution with an appreciable decrease in mass (8) occurs mainly in stars which belong to the first, early part of the main sequence. In this connection it is interesting to note that if the approximate Eq. (4) is the starting point of the calculations, the formal lower limit to the masses comes out less than that given by (17):

$$\mathfrak{M} > 0.1\mathfrak{M}_{\odot}. \tag{21}$$

However, it is precisely in the range of masses $(0.1-0.5)\cdot\mathfrak{M}_{\odot}$ that the theoretical mass-luminosity relationship derived from Eq. (4) deviates from the empirical relationship obtained for all the stars investigated (see [5], Fig. 2a). One should compare this with the results of Salpeter's investigations [12], in which he considers the form of the luminosity function for faint stars of the main sequence ($M > 3.5$; $\mathfrak{M} < 1.3\mathfrak{M}_{\odot}$) is determined by stellar evolution without mixing and without a change in mass. A similar conclusion about the nature of stellar evolution in the case of stars with a small mass (of the order of $\mathfrak{M}_{\odot}$ or less) was also reached by A. G. Masevich [13]. Finally, P. P. Parenago [14] even earlier came to the conclusion that the main sequence could be subdivided into two parts and this is also in agreement with the theory of the inner structure of stars.

Within the limits of error, the mass-luminosity relationship for the first part of the main sequence has the form of Eq. (14) [4] with

$$n = 4. \tag{22}$$

Substituting this expression into Eq. (8) and then integrating, we can easily calculate the age of the star considered if its initial mass $\mathfrak{M}_0$ is known:

$$\tau = \frac{\mathfrak{M}_{\odot}}{3kL_{\odot}}\left[\left(\frac{\mathfrak{M}_{\odot}}{\mathfrak{M}}\right)^3 - \left(\frac{\mathfrak{M}_{\odot}}{\mathfrak{M}_0}\right)^3\right]. \tag{23}$$

The stars belonging to the first part of the main sequence have different ages and are formed up to the present time [16, 17]. Therefore, the initial values of their mass, on the whole, must be comparatively large, since otherwise there would not be any distinct relation, even statistical, between the molecular weight and the mass of the star. Consequently, the second term on the right-hand side of Eq. (23) can be neglected in the first approximation if the very early stars are excluded:

$$\tau \approx \frac{\mathfrak{M}_{\odot}}{3kL}\left(\frac{\mathfrak{M}_{\odot}}{\mathfrak{M}}\right)^3. \tag{24}$$

Comparing this with the maximum possible age of the stars, which is usually taken to be the age of the galaxy or the age of the chemical elements [18]:

$$\max \tau \approx 5 \cdot 10^9 \text{ years}, \tag{25}$$

we find the corresponding value of the minimum possible mass:

$$\min \mathfrak{M} = \left(\frac{\mathfrak{M}_\odot}{3kL_\odot \max \tau}\right)^{1/3} \mathfrak{M}_\odot = 1.1\mathfrak{M}_\odot, \tag{26}$$

i.e., the sun itself is not one of the stars whose evolution is accompanied by a large loss of mass through corpuscular emission (8). This is in agreement with all the known estimates of the mass dissipation for the present-day sun; even with the most favorable assumptions, these estimates do not lead to a value of the mass ejected which is three orders of magnitude greater than the mass lost by radiation [19, 20]. Moreover, this is in agreement with the classification of the sun as belonging to the second part of the main sequence [21, 22].

3. As a first approximation, it is plausible to assume that during the time (25) in which the galaxy has been in existence, the stars under consideration were being formed at the same average rate:

$$\bar{n} = N/\max \tau, \tag{27}$$

and with the same distribution of initial bolometric magnitudes $\psi(M_0)$ ("initial luminosity function"). Then the present luminosity function for these stars is given by the obvious equation

$$\begin{aligned} N \int_M^\infty \varphi(M)\, dM = \bar{n} \max \tau \int_M^\infty \psi(M_0)\, dM_0 + \\ + \int_{-\infty}^{M} \bar{n}\left(\max \tau - \int_{M_0}^{M} \frac{dt}{dM}\, dM\right) \psi(M_0)\, dM_0, \end{aligned} \tag{28}$$

where M is not less than the magnitude which corresponds to the limit given by (26), i.e., stars of any initial magnitude are able to evolve during the time given by Eq. (25) and become stars of magnitude M. The first term on the right-hand side of this equation gives the number of stars, formed during the existence of the galaxy, whose magnitude from the start was less than M. The second term gives the number of stars, initially with a magnitude greater than M, whose magnitude during their lifetime becomes smaller than M. Differentiating Eq. (28) with respect to M, we get

$$N\varphi(M) = \left(\frac{dM}{dt}\right)^{-1} \bar{n} \int_{-\infty}^{M} \psi(M_0)\, dM_0. \tag{29}$$

If we assume that the stars considered enter the first part of the main sequence only with high luminosities,

$$\psi(M_0) = 0 \text{ for } M_0 > M^*, \tag{30}$$

then in the region of low luminosities ($M > M^*$), because of the normalization condition

$$\int_{-\infty}^{+\infty} \psi(M_0)\, dM_0 = 1, \tag{31}$$

the integral on the right-hand side of Eq. (29) becomes unity and the luminosity function takes the following form:

$$\varphi(M) = \frac{\bar{n}}{N}\left(\frac{dM}{dt}\right)^{-1} \quad \text{for} \quad M > M^*. \tag{32}$$

Expressing the luminosity and the mass in terms of the bolometric magnitude:

$$L / L_{\odot} = 10^{-0.4\,(M - M_{\odot})}, \tag{33}$$

$$\mathfrak{M} / \mathfrak{M}_{\odot} = (L / L_{\odot})^{1/4} = 10^{-0.1\,(M - M_{\odot})}, \tag{34}$$

and substituting these expressions into Eq. (8), we get

$$\frac{dM}{dt} = 10 \lg ek \frac{L_{\odot}}{\mathfrak{M}_{\odot}} 10^{-0.3\,(M - M_{\odot})}. \tag{35}$$

As a result the luminosity function for the region considered takes the following form:

$$\log \varphi (M) = C + 0.3\ M\ , \tag{36}$$

where the additive constant is completely defined on account of the Expressions (26) and (27):

$$\begin{aligned} c = \lg \frac{\mathfrak{M}_{\odot} \bar{n}}{10 \lg ek\, L_{\odot} N} - 0.3\, M_{\odot} \approx \lg \frac{\mathfrak{M}_{\odot}}{10 \lg ek\, L_{\odot}\, N \max \tau} - 0.3\, M_{\odot} \approx \\ \approx \lg \frac{3 \cdot 1.1^{3}}{10 \lg e} - 0.3\, M_{\odot} \approx -1.45, \end{aligned} \tag{37}$$

since

$$M_{\odot} = 4.7. \tag{38}$$

The mass given by Expression (26) corresponds to

$$\max M = 4.3. \tag{39}$$

With the values of the parameters obtained, the luminosity function for the stars under consideration is automatically normalized:

$$\int_{-\infty}^{\max M} \varphi (M)\, dM = 1. \tag{40}$$

P. P. Parenago [23] was the first to derive the theoretical Expression (36), to within an additive constant; in the derivation, he imposed for simplicity a more stringent restriction than that given by (30), namely, he assumed that all the stars are initially of the same spectral class ($\overline{M}_0 = M^*$), i.e., the initial luminosity function is a δ-function:

$$\psi (M_0) = \begin{cases} 0 & \text{for} \quad M_0 \neq \overline{M}_0, \\ \infty & \phantom{\text{for}} \quad M_0 = \overline{M}_0. \end{cases} \tag{41}$$

The values of log φ(M) obtained from observations lie reasonably close to the theoretical line [Eq. (36)] of slope 0.3, in the interval of bolometric magnitudes M from +5 to −3 [22,23], i.e.,

$$\overline{M}_0 \approx -3. \tag{42}$$

V. A. Ambartsumian [17] has considered that the deviation in the case of the brighter stars is due to the fact that the newly formed stars of the first part of the main sequence enter the sequence over an interval of absolute bolometric magnitudes and not only at one point. We can easily obtain the corresponding initial luminosity function for these stars by multiplying Eqs. (29) and (35) and then differentiating the product with respect to M:

$$\psi(M) = 10 \frac{kL_{\odot}N}{\mathfrak{M}_{\odot}{}^{n}} 10^{0.3M_{\odot}} 10^{-0.3M} \varphi(M) \left[\frac{d \lg \varphi}{dM} - 0.3\right] = \\ = K 10^{-0.3M} \varphi(M) \left[\frac{d \lg \varphi}{dM} - 0.3\right], \tag{43}$$

where the factor K can be calculated with the help of Eq. (37):

$$K = \frac{1}{\log e \cdot 10^{C}} \approx 65. \tag{44}$$

The normalization requirement is automatically fulfilled.

S. A. Kaplan [24] has derived Expression (43) to within a constant factor from an equation of the type (29), but this derivation was based on the assumption of the stationarity of the luminosity function and this assumption is by no means obvious. The number of stars under consideration and the interval of absolute bolometric magnitudes which contains these stars increase with the passage of time, i.e., the luminosity function, in general, cannot be considered stationary unless it is normalized in a fixed stellar-magnitude interval which is reached by any brighter star during the period when the processes of stellar evolution are of a stationary nature.

By substituting into Eq. (43) the luminosity function from [23], S. A. Kaplan obtained, for values of M_0 between −8 and −2, a bell-shaped distribution function $\psi(M_0)$ which is approximately symmetrical about the value

$$M_0 \approx -5. \tag{45}$$

With the help of P. P. Parenago's table [23], which gives the relative frequency distribution of absolute magnitudes for stars of various spectral classes, A. G. Masevich [1] has extracted from the general luminosity function the bolometric luminosity function of the main-sequence stars which is given (after a suitable renormalization) in Table 1; in the same table, the corresponding initial luminosity function $\psi(M_0)$, calculated from Eq. (43), is also given. Table 2 gives Salpeter's [12] luminosity function for the main-sequence stars; this function has also been converted to bolometric magnitudes. Despite the considerable difference between the two sets of initial data, which is due to some uncertainty in the luminosity function, the relative frequency of main-sequence stars, and the bolometric corrections, the general behavior of the two initial luminosity functions for the main-sequence stars are found to be similar, i.e., the distribution $\psi(M_0)$ has been determined with comparative certainty; moreover, in contrast to S. A. Kaplan's result [24], this distribution is found to exhibit two maxima, a smaller maximum situated near

$$\overline{M}_{01} = -4.5, \tag{46}$$

and a larger, main maximum near

$$\overline{M}_{02} = 0. \tag{47}$$

From the graph of the function $\psi(M_0)$ given in the figure, it is seen that the main portion of the newly formed stars enters the first part of the main sequence in the interval of absolute bolometric magnitudes M_0 between −3 and +1.5 [spectral type B4 − A4, $\mathfrak{M}_0 = (6-2)\,\mathfrak{M}_{\odot}$]. This is in very good agreement with the theoretical conclusion of A. I. Lebedinskii [25, 17, 26] that, because of gravitational instability (spontaneous division) there exists an upper critical limit to the mass of stars during the process of their formation and evolution:

$$\mathfrak{M}_k = \frac{25\mathfrak{M}_{\odot}}{\mu^2}. \tag{48}$$

From this point of view, the star which goes through the molecular-hydrogen stage ($\mu = 2$), as it is forming from the diffuse medium, cannot have an initial mass $\mathfrak{M}_0$ greater than $6\mathfrak{M}_{\odot}$. According to A. I. Lebedinskii and L. E. Gurevich [17, 26, 27], the more massive stars are formed by the conversion of dwarfs into giants (through the absorption of the diffuse matter by stars already in existence), thus by-passing the molecular-hydrogen stage. The author has pointed out [28] that if this idea is correct, then there must be a "break" in the luminosity function

TABLE 1

M	$\lg\varphi+10$	φ	$\frac{d\lg\varphi}{dM}-0.3$	ψ
−6	6.01	$1.0\cdot10^{-4}$	0.04	0.02
−5	6.35	$2.2\cdot10^{-4}$	0.06±0.02	0.03±0.01
−4	6.73	$5.4\cdot10^{-4}$	0.10±0.02	0.06±0.01
−3	7.15	$1.4\cdot10^{-3}$	0.02±0.10	0.01±0.05
−2	7.37	$2.3\cdot10^{-3}$	0.03±0.11	0.02±0.08
−1	7.81	$6.4\cdot10^{-3}$	0.19±0.05	0.16±0.04
0	8.35	$2.2\cdot10^{-2}$	0.22±0.02	0.32±0.03
1	8.85	$7.1\cdot10^{-2}$	0.13±0.07	0.30±0.15
2	9.21	$1.0\cdot10^{-1}$	−0.02±0.08	−0.05±0.20
3	9.41	$2.6\cdot10^{-1}$	0.00±0.10	0.00±0.21
4	9.80	$6.3\cdot10^{-1}$	0.01±0.08	0.03±0.24
5	10.03			

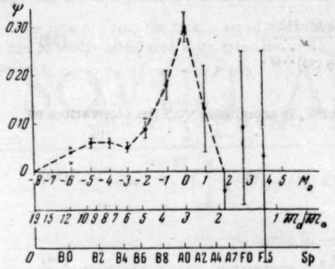
TABLE 2

M	$\lg\varphi+10$	φ	$\frac{d\lg\varphi}{dM}-0.3$	ψ
−6	5.65	$4.5\cdot10^{-5}$	0.28	0.05
−5	6.23	$1.7\cdot10^{-4}$	0.24±0.04	0.08±0.01
−4	6.72	$5.2\cdot10^{-4}$	0.13±0.06	0.07±0.03
−3	7.09	$1.2\cdot10^{-3}$	0.08±0.02	0.05±0.01
−2	7.49	$3.1\cdot10^{-3}$	0.12±0.02	0.10±0.02
−1	7.94	$8.7\cdot10^{-3}$	0.18±0.04	0.19±0.04
0	8.46	$2.9\cdot10^{-2}$	0.14±0.08	0.27±0.15
1	8.83	$6.8\cdot10^{-2}$	0.01±0.06	0.02±0.12
2	9.08	$1.2\cdot10^{-1}$	−0.02±0.02	−0.04±0.04
3	9.38	$2.4\cdot10^{-1}$	0.09±0.09	0.18±0.18
4	9.86	$7.2\cdot10^{-1}$	0.00±0.18	0.00±0.54
5	9.99			

TABLE 3

M	−6	−5	−4	−3	−2	−1	0	1	2	3	4
ψ	0.04	0.06	0.06	0.05	0.09	0.18	0.30	0,13	0	0	0
φ Cygnus	0.00	0.00	0.00	0.10	0.12	0.12	0.25	0.13	0.10	0.16	0
φ Orion	0.02	0.10	0.09	0.05	0.03	0.35	0.17	0.08	0.01	0.00	0.00

at a value of the luminosity corresponding to the transition to the giants. This expected feature is plainly seen in Fig. 1: the region of the giants, which extends from $M = -3$ to $M = -8$ where the function $\psi(M_0)$ is practically zero, is clearly separated from the main part of the luminosity function. Further, at about $M_0=2$ there is an indication, although the errors are large, that the sign of the function ψ changes, i.e., an indication that the corresponding stars have left this part of the main sequence (their conversion into giants?). Contrary to A. G. Masevich's result [1], where the quantity $\overline{n}$ was negative as a result of a simple mistake [29], the function $\psi(M_0)$ can, in general, be negative, provided that the normalization condition given by Eq. (31) is satisfied. But in fact, of course, we are not obliged to consider that the function ψ changes sign and we should assume simply that for $M_0=2$ it becomes zero. As regards the conversion of dwarfs into giants through accretion, we must suppose that this takes place, taking into account the considerations given in [28], not in a chance encounter between a dwarf star and an interstellar gas cloud, but deep inside the primordial gas cloud, relative to which the



star had a sufficiently small velocity (less than parabolic) right from the start. If we assume that the average bolometric magnitude of the newly-formed stars must be close to the most probable value $\overline{M}_{02}$, then we should consider that the asymmetry of the function $\psi(M_0)$ is due to the movement of stars from the right-hand side of the curve into the secondary maximum on the left, i.e., most of the young dwarfs are transformed into giants even before they leave the primordial gas cloud. This results, apparently, in the characteristic form of the function $\psi(M_0)$ found above.

4. If the stellar associations are systems consisting of young stars, then the investigation of their luminosity function must provide the most direct information on the form of the function $\psi(M_0)$.

In the case of the Cygnus association, V. A. Ambartsumian [17] has derived the luminosity function from the star counts of Miller, who investigated the association and a normal region of the Milky Way in Cygnus. Converted to bolometric magnitudes by means of P. P. Parenago's standard table [22], this function is compared in Table 3 with the mean weighted distribution $\psi(M_0)$ taken from the figure. The same figure also gives the luminosity function obtained by P. P. Parenago [30] for the main-sequence stars of the Orion association. Both observed luminosity functions for the stellar associations, as was to be expected, are fairly close to the theoretical initial luminosity function ψ.

5. The luminosity functions for the open stellar clusters have a different appearance [31]. Their noticeable displacement in the direction of lower luminosities apparently points to the considerable age of clusters by comparison with that of associations. Their slower rise with increasing luminosity, as compared to the general luminosity function, is apparently due to the destruction of the clusters, during which the number of stars with a given mass that escape increases with decreasing mass [32, 22].

6. An independent confirmation of the conclusion that stellar evolution along the first part of the main sequence progresses in the direction from hot giants to dwarfs is the fact of a corresponding increase in the dispersion of the peculiar velocities. Osterbrock [33] has confirmed the result of Spitzer and Schwarzschild [34], who found that the dispersion in the stellar peculiar velocities can increase appreciably in $3 \cdot 10^9$ years as the result of gravitational perturbation by the galactic matter with density fluctuations. According to Spitzer and Schwarzschild, during the period $\overline{\tau}$ of its existence any subsystem consisting of Population I stars will show an increase in the dispersion of the peculiar velocities in the galactic plane from the initial value V_0 to

$$V\,(\overline{\tau}) = V_0\,(1 + \overline{\tau}/t_E)^{1/3}, \tag{49}$$

where

$$t_E = \frac{4V^3{}_0}{3\pi^{3/2} \cdot G^2 n_c m_c^2 \ln\alpha}\,; \tag{50}$$

here G is the gravitational constant, $\ln\alpha \simeq 3$, m_c is the mass of the perturbing bodies, and n_c is their number per unit volume. In this case, the more massive galactic inhomogeneities [35] will have the dominant role among the effective perturbing bodies.

The age of an individual star belonging to the first part of the main sequence is given by Eq. (23) which, with the help of Eq. (34), can be written as

$$\tau = \frac{\mathfrak{M}_\odot}{3kL_\odot}\,10^{-0.3M_\odot}\,(10^{0.3M} - 10^{0.3M_0}). \tag{51}$$

The average age of the stars whose bolometric magnitudes lie between M_1 and M_2 is given by the obvious expression

$$\bar{\tau} = \int_{M_1}^{M_2} \left[\frac{\int_{-\infty}^{M} \tau\psi(M_0)\,dM_0}{\int_{-\infty}^{M} \psi(M_0)\,dM_0} \right] \frac{\varphi(M)\,dM}{\int_{M_1}^{M_2} \varphi(M)\,dM}\,. \tag{52}$$

If we substitute this expression into the theoretical Eq. (49) and assume, in accordance with the observations on the earliest main-sequence stars [36], that

$$V_0 = 10 \text{ km/sec}\,, \tag{53}$$

we can easily find that for

$$t_E = 3.2 \cdot 10^8 \text{ years}\,, \tag{54}$$

the theory is in excellent agreement with the observed mean peculiar-velocity dispersions obtained by Ia. E. Einasto [37] for the main-sequence stars of various spectral classes up to F7. This comparison is made in Table 4; the theoretical Expression (36) for the luminosity function was used to calculate $\varphi(M)$ from (52); the observed dispersions of the stellar peculiar velocities in the galactic plane were calculated from the dispersions σ_m which are proportional to the former and which correspond to the minor axis of the biaxial velocity ellipsoid [38, 37]:

$$V = \left(\frac{1}{2}\sigma_2^2 + \frac{1}{2}\sigma_3^2\right)^{1/2} = \left(\frac{1}{2}0.67^2 + \frac{1}{2}\right)^{1/2}\sigma_3 = 0.85\sigma_3 = \frac{0.85}{0.60}\sigma_m = 1.4\sigma_m\,. \tag{55}$$

As was to be expected, the later classes are to a large extent inhomogeneous in the peculiar-velocity dispersion and age [37].

When we substitute the values of t_E, given by (54), and V_0, given by (53), into Eq. (50) and take the average density of the nonuniformly distributed galactic matter to be [34]

$$n_c m_c \approx 3 \cdot 10^{-24} \text{ g/cm}^3, \tag{56}$$

we find that the mass of the effective inhomogeneities is

$$m_c \approx 3 \cdot 10^5 \mathfrak{M}_\odot, \tag{57}$$

which is in good agreement with the masses of such galactic formations as the globular clusters [39-41], stellar groups which agitate the interstellar medium [17, 26], the interstellar-cloud aggregates [42], and the Orion system [30, 43].

TABLE 4

Sp		Oe5 — B5	A5 — A9	F0 — F4	F5 — F7
$M_1 \div M_2$		<-2.6	$1.6\div2.8$	$2.8\div3.9$	$3.9\div4.35$
$\bar{\tau}$, years		$<4\cdot10^{-7}$	$1.0\cdot10^9$	$2.5\cdot10^9$	$4.2\cdot10^9$
V, km/sec	Theory	10	16	20	24
	Observation	10 [36]	17±1	20±1	24±1

7. The fact that stars belonging to the first part of the main sequence evolve by corpuscular emission is directly associated with the observation that the process of their formation has continued up to the present time: if $\underline{n}$, the rate of star formation, were to become greater than the average, $\bar{n}$, the primordial diffuse medium would be rapidly consumed, the rate of star formation would decrease sharply until the corpuscular emission from

existing stars provides a further supply of the diffuse matter. Therefore, our assumption that the star formation through such a self-regulating process remains constant is reasonable.

The rate at which the total mass of stars already in existence decreases as the result of corpuscular emission can be easily calculated if we make use of Eq. (29) and change the order of integration in the resulting equation:

$$\begin{aligned}\frac{\partial m'_{\mathrm{st}}}{\partial t} &= N \int\limits_{-\infty}^{\max M} \frac{d\mathfrak{M}}{dt}\varphi(M)\,dM = \bar{n}\int\limits_{-\infty}^{\max M}\left[\frac{d\mathfrak{M}}{dt}\left(\frac{dM}{dt}\right)^{-1}\int\limits_{-\infty}^{M}\psi(M_0)\,dM_0\right]dM = \\ &= \bar{n}\int\limits_{\infty}^{\min \mathfrak{M}}\left[\int\limits_{-\infty}^{M}\psi(M_0)\,dM_0\right]d\mathfrak{M} = \bar{n}\int\limits_{-\infty}^{\max M}\left[\int\limits_{\mathfrak{M}_0}^{\min\mathfrak{M}} d\mathfrak{M}\right]\psi(M_0)\,dM_0 = \\ &= \bar{n}\min\mathfrak{M} - \bar{n}\int\limits_{-\infty}^{\max M}\mathfrak{M}_0\psi(M_0)\,dM_0.\end{aligned} \tag{58}$$

On the other hand, the rate at which the total mass of stellar matter increases as the result of newly forming stars is given by

$$\frac{\partial m''_{\mathrm{st}}}{\partial t} = \bar{n}\int\limits_{-\infty}^{\max M}\mathfrak{M}_0\psi(M_0)\,dM_0 = \bar{n}\overline{\mathfrak{M}}_0. \tag{59}$$

As a result, there is a general increase in the total mass of stars belonging to the first part of the main sequence:

$$\frac{dm_{\mathrm{st}}}{dt} = \bar{n}\min\mathfrak{M}, \tag{60}$$

but at an appreciably lower rate, since at the present time min $\mathfrak{M}$ [Eq. (26)], the minimum mass of a typical star belonging to the first part of the main sequence, is approximately one fifth as large as the average initial mass:

$$\overline{\mathfrak{M}}_0 \approx 4.7\,\mathfrak{M}_\odot. \tag{61}$$

During the period of existence of the galaxy, the total mass of stars which go through the evolutionary stage under consideration is

$$m = \bar{n}\overline{\mathfrak{M}}_0\max\tau. \tag{62}$$

The total mass of stars that, at the present time, belong to the first part of the main sequence can be calculated from Eq. (60); taking Expression (26) into account and integrating Eq. (60), we get

$$\begin{aligned} m_{\mathrm{st}} &= \int\limits_0^{\max\tau}\frac{dm_{\mathrm{st}}(t)}{dt}\,dt = \int\limits_0^{\max\tau}\bar{n}\min\mathfrak{M}(t)\,dt = \\ &= \int\limits_0^{\max\tau}\bar{n}\min\mathfrak{M}(\max\tau)\left[\frac{\max\tau}{t}\right]^{1/3}dt = \frac{3}{2}\bar{n}\min\mathfrak{M}\max\tau.\end{aligned} \tag{63}$$

Consequently, the mass m − m_{st} lost by the stars is comparable to m and m_{st}. Obviously, this means that the stationarity of the star-formation process has already been established, while the masses of the primordial diffuse medium and of the stars under consideration are still comparable in magnitude.

Let us estimate the total amount of helium which has been produced from hydrogen during the evolution of the stars belonging to the first part of the main sequence. The total energy radiated by these stars during the period of existence of the galaxy is

$$E = \int\limits_{0}^{\max\tau} N(\tau)\,\overline{L}(\tau)\,d\tau, \tag{64}$$

where

$$N(\tau) = \overline{n}\tau, \tag{65}$$

$$\begin{aligned}
\overline{L}(\tau) &= \int\limits_{-\infty}^{\max M(\tau)} L\varphi_{\tau}(M)\,dM = \\
&= \int\limits_{-\infty}^{\max M(\tau)} \left[L_{\odot}\cdot 10^{-0.4(M-M_{\odot})}\,\frac{\overline{n}}{N(\tau)}\left(\frac{dM}{dt}\right)^{-1}\int\limits_{-\infty}^{M}\varphi(M_0)\,dM_0\right]dM = \\
&= \frac{\mathfrak{M}_{\odot}}{10\lg ek\tau}\,10^{0.1M_{\odot}}\int\limits_{-\infty}^{\max M(\tau)}\left[10^{-0.1M}\int\limits_{-\infty}^{M}\psi(M_0)\,dM_0\right]dM = \\
&= \frac{\mathfrak{M}_{\odot}}{10\lg ek\tau}\,10^{0.1M_{\odot}}\int\limits_{-\infty}^{\max M(\tau)}\left[\psi(M_0)\int\limits_{M_0}^{\max M(\tau)}10^{-0.1M}\,dM\right]dM_0 = \\
&= \frac{\mathfrak{M}_{\odot}}{k\tau}\,10^{0.1M_{\odot}}\int\limits_{-\infty}^{\max M(\tau)}[10^{-0.1M_0} - 10^{-0.1\max M(\tau)}]\,\psi(M_0)\,dM_0 = \\
&\doteq \frac{\mathfrak{M}_{\odot}}{k\tau}\,10^{0.1M_{\odot}}\int\limits_{-\infty}^{+\infty}10^{-0.1M_0}\psi(M_0)\,dM_0 = 1.5\,\frac{\mathfrak{M}_{\odot}}{k\tau}\,10^{0.1M_{\odot}};
\end{aligned} \tag{66}$$

in the last estimate, values of τ that are too small have been neglected since the process of star formation can hardly be considered to have been stationary at that time. The result is:

$$E \approx 1.5\,\frac{\mathfrak{M}_{\odot}}{k}\,10^{0.1M_{\odot}}\,\overline{n}\max\tau. \tag{67}$$

This amount of energy is liberated in the conversion of a mass

$$m^{*} = \frac{E}{\alpha} = 1.5\,\frac{\mathfrak{M}_{\odot}}{k\alpha}\,10^{0.1M_{\odot}}\,\overline{n}\max\tau = 0.8\mathfrak{M}_{\odot}\overline{n}\max\tau \tag{68}$$

of hydrogen into helium. Consequently, the minimum average abundance of helium in galactic matter, which has passed or is passing through the stage corresponding to the first part of the main sequence, must be

$$Y = m^{*}/m = 0.17 \tag{69}$$

(17% of the mass, i.e., approximately 5% of the number of atoms if we neglect the presence of other elements, in addition to hydrogen, the relative abundances of which are in general small).

The theoretical estimate obtained is very realistic. Aller [44], from an investigation of the spectra of six B-stars (φ Orionis, χ^2 Orionis, ϵ Canis Majoris, i Herculis, δ Ceti, and γ Pegasi), found that for these stars the average abundance of helium is 5% by number of atoms or 16% by mass. Data on the relative abundance of helium in interstellar space are virtually absent [45], but its presence in some gaseous nebulae (NGC1976, NGC-2237, and NGC6523) has already been established directly [46].

8. There are many arguments in favor of the conclusion that the components of multiple stars must, as a rule, have the same age. Therefore, the association of similar stars belonging to the second part of the main

sequence with various stars belonging to the first part of the main sequence points to a considerable spread in the ages of the stars of later spectral classes. This is also in agreement with the kinematic inhomogeneities of these stars whose peculiar-velocity distribution cannot be represented by only one gaussian [37]. Isolating from the total number of M-dwarfs those that belong to Population II (with a dispersion V = 56 km/sec), Spitzer and Schwarzschild [47] came to the conclusion that Population I stars belonging to the second part of the main sequence have a constant dispersion of peculiar velocities in the galactic plane,

$$V = 20 \text{ km/sec}, \tag{70}$$

irrespective of their spectral class. The corresponding theoretical estimate can be obtained on the assumption that these stars have been born at the same rate throughout the lifetime of the galaxy:

$$\begin{aligned}\bar{V} = \Bigg[\int\limits_0^{\max\tau} V^2 \frac{d\tau}{\max\tau}\Bigg]^{1/2} = V_0\Bigg[\int\limits_0^{\max\tau}\left(1+\frac{\tau}{t_E}\right)^{2/3}\frac{d\tau}{\max\tau}\Bigg]^{1/2} = \\ = V_0\left[\frac{(1+\max\tau/t)^{5/3}-\max\tau/t_E}{\frac{5}{3}\max\tau/t_E}\right]^{1/2} \approx 19 \text{ km/sec}.\end{aligned} \tag{71}$$

9. A. I. Lebedinskii [17, 26] considers that the corpuscular emission from stars is associated with rapid rotation and with the presence of strong and variable magnetic fields ("spots"). All of these properties are characteristic of typical stars of early spectral classes, but are absent in stars belonging to the second part of the main sequence. Therefore, it is natural to investigate the evolution of the latter as taking place without a loss of mass and without intermixing between the core and the envelope. Evolving in this way, the star after the conversion of 12% of its mass from hydrogen into helium (Schonberg-Chandrasekhar limit) changes in structure and rapidly leaves the main sequence into the region of red giants [48]. In the case of stars with M ≈ 3.5, this process in fact continues for about 5 billion years. According to Sandage and Schwarzschild [48], this is precisely the explanation of the absence of very bright main-sequence stars in globular clusters which were formed during the formation of the galaxy itself (max τ years ago). Using a similar method, Salpeter has attempted [12], to explain the sharp decrease of the luminosity function for the stars belonging to the early part of the main sequence on the assumption that the corresponding Population I stars are born at the same rate throughout the lifetime of the galaxy but that not all of them remain on the main sequence during their lifetimes. However, the initial luminosity and mass distributions of the stars calculated by him exhibit unsubstantiated irregularities, which he has ignored, in the region of bolometric magnitudes M between +3 and 0. Therefore, it is reasonable to think that this type of evolutionary behavior can only be characteristic of a small fraction of the stars belonging to the early spectral classes, for example, stars with an anomalously slow axial rotation. If the red giants of Population I are formed from such main-sequence stars, then their age varies between the average age of the stars belonging to the second part of the main sequence and max τ, i.e., the dispersion of their peculiar velocities in the galactic plane must lie between the limits V ≈ 19 km/sec (71) and V(max τ) ≈ 25 km/sec. The observed values of the dispersion [49, 50] are in excellent agreement with this: V = 21-25 km/sec [47].

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STATISTICAL MECHANICS OF THE SIMPLEST TYPES OF GALAXIES

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In accordance with the principles given by the author in another paper* quasi-stationary stellar systems are considered. These have finite phase volumes and times of relaxation which are not longer than the characteristic time-scales (of the order of magnitude of the rotational period). In these systems the actual phase distribution cannot differ appreciably from the most probable distribution.

In order to find the most probable phase distribution the author applies the method of "phase cells," (known in statistical physics) together with the method of additive parameters. Every stellar system, from the point of view of a dynamical system of material particles, is characterized first of all by three additive parameters expressing properties of motion: the mass — M, the sum of the energies of motion of the stars — E, and the sum of the rotational moments of the stars — K. Every stellar system completely defined by the values of these three parameters is called dynamically definable or briefly a D-system. These are systems of the simplest type, as generally three dynamic parameters are not sufficient for uniquely defining a stellar system (e.g. our galaxy).

In the present paper only D-systems are considered. It is shown that the following characterize the most probable state of D-systems:

1) Uniform rotation (Theorem I); 2) "isothermal" Maxwellian distribution of peculiar stellar velocities (Theorem II). However, Maxwell's law is valid only within a limited range of velocities.

The application of Poisson's equation together with Theorems I and II leads to the conclusion that in the first approximation the stellar density is constant throughout the main body of the D-galaxy and is proportional to the square of the angular velocity of rotation (Theorem III). D-galaxies appear to be dynamically unstable as they do not comply with the requirements of Poincare's criterion for the upper limit of angular velocity (Theorem IV).

Nevertheless we can associate with every D-system a certain classical equilibrium figure of a uniform, gravitating liquid mass. Maclaurin's ellipsoids of the 1st type (planetary) correspond to E-galaxies and to the nuclei of normal S-spirals. Maclaurin's ellipsoids of the 2nd type (disk-shaped) correspond to SO - and normal S-spirals. Finally, the prolate ellipsoids of Jacobi correspond to a new type of galaxies which we shall term needle-shaped (acicular), or simply A-galaxies.

These galaxies are very long and very thin bodies. They are observed in sufficient numbers in the skies but ordinarily they are confused with genuine normal spirals seen edge-on. Their presence among edge-on spirals is revealed by the apparent excess of the number of such objects as compared to theoretical predictions based on the assumption that all these objects are disk-shaped. Such an excess was pointed out as early as 1920-22 by J. H. Reynolds [5]. The main features of A-galaxies are: a very tufty appearance with an absence of any regular structure (nuclei, spiral arms, annular structure and the like). They represent, probably, one of the earliest stages of galactic evolution. Typical representatives of the A-class of galaxies are: NGC 2188, 2796, 3034, 3556.

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Owing to their great instability, the A-galaxies are observed rather seldom in their pure form. In the majority of cases they form the main bodies of SB-spirals displaying a state of disruption through an outflow of streams of stellar gas from their outermost points. (NGC 7741 is a typical example.)

Pear-shaped asymmetry, theoretically predicted for liquid masses by G. Darwin, H. Poincare and A. Liapunov, is also possible for galaxies. It is actually observed in many instances (e. g. NGC 4455, 4631, 7479). In the extreme cases the asymmetry leads to the formation of one-arm spirals (e.g. NGC 4038, 4254).

In the second approximation a differential equation of the Helmholtz type for the additional star density was derived. For its solution a suitable boundary problem based on the uniform equilibrium body of the first approximation must be formulated. In the simplest case of a very thin and flat disk it is possible to apply the Fourier method of separating variables. This leads to a Bessel equation for the distribution of the additional star density along the radius. Different particular solutions correspond either to the central nucleus or to a system of concentric rings. A "barometric" formula was obtained for the density distribution in the z-direction. In the azimuthal direction a harmonic solution was found. Mathematically this solution is identical with the well-known solution for an elastic round membrane.

The above solution permits the interpretation, at least qualitatively, of a number of different structural details of SO- and SB-spirals, which have an annular structure and a strong condensation of mass towards the center. Owing to low density in the end parts of the main body, which is now less prolate, the outflow of stellar gas goes on slower and the streams are thinner than in the case of "classical" SB-spirals, such as NGC 7741 with no central condensation. (Typical examples: NGC 488, 1398, 4725.) The existence of the solution of the Helmholtz equation ensures the stability of the central nucleus and of the annular pattern and prevents their dissipation. At the same time the existence of a flat Maclaurin ellipsoid of equilibrium prevents the majority of stars, ejected from other parts of the old prolate main body, from dissipating and permits them to form a new disk-shaped body.

In all the above cases the spiral arms must be trailing.

1. Introduction

On the basis of the ideas presented in another paper* we will consider that in every star system there exists an inner part, or the main body, and an outer part, or the halo. The character of a star system is determined by the structure of its main body. The halo surrounds the main body in the form of a structureless star cloud.

The dynamical theory must first of all describe the equilibrium shape of the main body of a star system. Then the structure of the halo can be determined by obtaining a suitable function to describe the distribution of the orbit elements of the individual stars moving in a gravitational field which depends almost entirely on the attraction of the main body.

The phase density has been determined only inside the main body [I].

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* See the footnote to the abstract. In the following we will refer to it by the symbol [I].

We will assume that the relaxation time T_0 is considerably less than the time necessary for a star, moving with a velocity $\underline{v}$ equal to the mean value of the residual velocities, to cross a given star system along its diameter D[*]:

$$T_0 < D/v. \tag{1}$$

Doing this, we assume that, at least inside the main body, there exists an efficient system of irregular forces, analogous to viscosity in gases, which enables us to consider the star system as a continuous medium and, consequently, to use the Boltzmann equation and the equations of stellar hydrodynamics. On the other hand, because of the action of irregular forces, the phase distribution must differ little from the most probable one for the given star system. Only by combining two methods, statistical and hydrodynamical, are we able to eliminate the indeterminacy in the solution of the dynamical problem.

Observations show that the distinguishing feature of galaxies is their axial rotation which is either rigid-body rotation, as in E- and, apparently, SB-type galaxies, or quasi-rigid-body rotation as in S-type galaxies, in the inner part up to the distance corresponding to the maximum velocity of rotation (for this see [I]).

We will assume that the galaxies investigated by us, firstly, are isolated dynamical systems; secondly, we will assume that they are in a quasi-stationary state, by which we mean that the star system is in a state of dynamical equilibrium at every instant of time. With dynamical equilibrium the phase density and, consequently, all the internal physical parameters of the galaxies can be considered to be explicitly independent of time.[**] We will not consider any interaction forces apart from gravitational forces. Although we have sufficient data to accept the existence of magnetic fields in certain types of galaxies (for example, in ours), as yet it is insufficiently clear what is the role played by the fields in galactic dynamics. It is equally uncertain whether their action is the same in all types of galaxies. It appears that they are only important in gigantic spirals of the type of our galaxy and, even there, their role is probably limited to the influence they have on the distribution of gas and dust. As regards stellar motion, it is not affected by magnetic fields.

2. The Method of Additive Parameters

According to Jeans' Theorem, the phase density f must be a function of only the integrals of motion of a free point. For example, in the case of a star system whose mass distribution is rotationally symmetric[***] there will be two such integrals: the energy integral and the angular momentum integral ("integral of areas"). In cylindrical coordinates they will have the form

$$I_1 = \mathrm{P}^2 + \Theta^2 + Z^2 - 2U(\rho,z); \quad J_2 = \rho\Theta; \tag{2}$$

where $U(\rho, z)$ is the gravitational potential.

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[*] For example, for our galaxy we can take D = 16 kps, v = 20 km/sec and then we find $T_0 < 4 \cdot 10^8$ years.

[**] By investigating equilibrium bodies we also take the necessary first step towards the study of nonstationary galaxies. The nonstationary motion can be regarded as due to the breakdown of initial stationary conditions. For example, we can consider SB-spirals to be formed by the disruption of a primary elongated cylindrical or ellipsoidal body by an escape of a gas of stars from its ends. In the same way, we can consider the formation of dark bands along the equator of E-galaxies to be due to the disruption of an ellipsoidal body. In both cases it is natural to assume that the disruption is produced by the centrifugal force due to rotation.

[***] In the following we will distinguish between the rotational and axial symmetry of a function. In the former case, the function depends only on ρ, the distance from the axis of symmetry, and the coordinate $\underline{z}$. In the latter case, each point (ρ, θ, z) has a point $(\rho, \theta + \pi, z)$ which is situated symmetrically with respect to the axis. The level surfaces of the function will be surfaces of rotation in the first case, but not in the second. A spheroid is an example of a function with rotational symmetry, an ellipsoid that of a function with axial symmetry.

Therefore,

$$f = F(I_1, I_2), \tag{3}$$

where F is an arbitrary function.

On the other hand, with the condition that in a star system there exists an efficient mechanism of relaxation in the form of irregular forces, the phase density must differ little from the most probable distribution for the star system with the given physical characteristics.

In the method of additive parameters we form functions $\varphi(I_1, I_2)$ of the integrals so that their arithmetic sum for all the stars of the given system represents an additive parameter, i.e., a quantity which describes some characteristic property of this system.

Let

$$P_1 = \sum_{i=1}^{N} m_i\varphi_1(I_{1i}, I_{2i}); \quad P_2 = \sum_{i=1}^{N} m_i\varphi_2(I_{1i}, I_{2i}); \ldots; P_n = \sum_{i=1}^{N} m_i\varphi_n(I_{1i}, I_{2i}) \tag{4}$$

be a system of additive parameters satisfying the conditions that: (1) they are independent and (2) that they determine uniquely the star system. Here I_{1i}, I_{2i}, m_i denote the values of the integrals and the mass of the i-th star, $\varphi_1, \varphi_2, \ldots, \varphi_n$ are the characteristic functions obtained by the method described above.

Any system of additive parameters which satisfies conditions (1) and (2), we will call a defining system. Each star system, generally speaking, has an infinite number of equivalent defining systems of parameters. This is seen, if only from the fact that any linear combination of additive parameters of a given system is itself an additive parameter.

Having found a defining system of additive parameters of the type (4), we then look for the most probable phase distribution satisfying the condition that the parameters $P_1, P_2, \ldots, P_n$ have the specified values. In this case, as we will see below, Jeans' Theorem is automatically satisfied. The method of additive parameters, obviously, can be extended to any number of independent integrals of motion (greater or less than two).

The simplest type of star systems is the dynamically-definable systems or, briefly, D-systems. We can arrive at an understanding of dynamically-definable systems by the following method.

The simplest additive parameters are

$$1)\ P_1 = \sum_{i=1}^{N} m_i; \quad 2)\ P_2 = \sum_{i=1}^{N} m_i I_{1i}; \quad 3)\ P_3 = \sum_{i=1}^{N} m_i I_{2i}. \tag{5}$$

We will show that these parameters have constant values as the result of the general theorems for an isolated system of mass points. In fact, the physical meaning of P_1 is obvious: it is equal to the mass of the star system. Therefore, the invariance of P_1 is an expression of the law of mass conservation. We will call it the mass parameter and will denote it in the following by the symbol M. The second parameter is equal to the sum of the kinetic energies of the individual stars moving in the total gravitational field of the star system. It has the expanded form:

$$P_2 = \sum_{i=1}^{N} m_i (\mathrm{P}_i^2 + \Theta_i^2 + Z_i^2 - 2U_i), \tag{6}$$

where we have written for brevity

$$U_i = U(\rho_i, z_i) = G \sum_{k=1}^{N}{}' \frac{m_k}{r_{ik}}, \tag{7}$$

where G is the gravitational constant, r_{ik} denotes the distance between the k-th and i-th stars, and the prime in the summation sign means that the term for k = i has been omitted in it. From this it is seen that

$$\sum_{i=1}^{N} m_i U_i = -2W, \tag{8}$$

i.e., twice the potential energy of the system (with opposite sign) since in the summation each star is taken into account twice. As regards the sum

$$\sum_{i=1}^{N} m_i (P_i^2 + \Theta_i^2 + Z_i^2) = 2T, \tag{9}$$

it is equal to twice the kinetic energy of the system. Therefore,

$$P_2 = 2(T + 2W). \tag{10}$$

Even if P_2 is not equal to twice the total energy of the system, it is constant because in the case of a quasi-stationary system of mass points, not only is the law of conservation of energy obeyed, i.e., T + W = constant, but also the virial theorem according to which 2T + W is zero. Therefore, in a quasi-stationary system the kinetic and the potential energies are constant separately, but only their combination P_2 has the additive property with respect to the integrals of motion of the individual stars. We will call P_2 the energy parameter and in the following we will denote it by the symbol E.

Finally, P_3 is the sum of the rotational moments of the individual stars about the axis of symmetry of the star system. In the following we will restrict ourselves to the investigation of only the star systems rotating about the axis of symmetry.* Then P_3 will be constant as the result of the law of conservation of angular momentum. In the following we will denote P_3 by the symbol K. We will refer to M, E and K as dynamical parameters.

In general, the dynamical parameters do not form a defining system, i.e., they are insufficient for a unique description of a given type of star system. The general theorems of mechanics, however, do not give any integrals, except the energy and momentum integrals, for an arbitrary isolated system of mass points.** From this follows that all the other additive parameters must be nondynamical, i.e., they must be statistical in nature. We can thus divide star systems into two classes: dynamically definable and dynamically nondefinable systems. It is clear that every star system among its defining parameters must, in any case, have three dynamical parameters. Therefore, from the point of view of statistical mechanics, dynamically definable star systems are systems of the simplest type and for this reason we will at first restrict ourselves to the investigation of such systems only, leaving the investigation of more complicated dynamically nondefinable systems to a future paper.

3. The Most Probable Phase Distribution for the Simplest Type of Galaxies

We will investigate a D-system of stars (a galaxy) which is defined by three additive dynamical parameters: the mass M, the energy E, and the angular momentum K. We will consider that the system is rotating about its axis of rotational symmetry. For simplicity, we will assume that all the stellar masses are equal. Then, denoting by N the number of stars in the galaxy, we will have three parameters:

$$N,\ E = \sum_{i=1}^{N} (P_i^2 + \Theta^2 + Z_i^2 - 2U_i);\quad K = \sum_{i=1}^{N} \rho_i \Theta_i. \tag{11}$$

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*In other words, we will not investigate the cases in which, in addition to pure rotation, there are precessional and nutational motions, since these are unlikely to arise in a star system not subject to the influence of external forces.

**We do not, of course, include the integral which describes the uniform motion of the center of mass as it does not enter into our calculations.

Taking into account that the main body (which we have in mind here) of the galaxy occupies a finite volume, while the velocities are limited by the escape velocity, we conclude that the phase volume of the galaxy Γ is limited. To find the most probable phase distribution we will use the method of phase cells, which is well-known in physics. For this we divide the phase volume into a very large number m of very small phase cells which, however, must satisfy the condition

$$1 \ll m \ll N, \tag{12}$$

i.e., m must be very large and, at the same time, very small by comparison with N. In other words, the average number of stars in one phase cell N/ m must be larger than unity (for example, several tens or several hundred). We denote the volume of the k-th cell by γ_k. γ_k satisfies the relation

$$\sum_{k=1}^{m} \gamma_k = \Gamma. \tag{13}$$

Let us now assume, as is usual in such cases, that the a priori probability for any star to lie in the k-th cell is the same for all stars, is independent of the position of the cell in the phase volume Γ, and is equal to the geometrical probability

$$p_k = \gamma_k / \Gamma. \tag{14}$$

Obviously, this hypothesis is equivalent to the assumption that in the absence of the constraints imposed by the conditions E = constant and K = constant, the phase distribution will be uniform over the whole volume Γ.

The probability that with a random distribution of stars there will be $n_1, n_2, \ldots, n_m$ stars in the cells is given by the binomial distribution as

$$P = \frac{N!}{n_1!\, n_2! \ldots n_m!}\, p_1^{n_1},\, p_2^{n_2} \ldots p_m^{n_m}. \tag{15}$$

Within each cell we can take the phase coordinates of the points to be equal. Hence, the quantities given by (11) can be rewritten in the form

$$N = \sum_{k=1}^{m} n_k; \quad E = \sum_{k=1}^{m} n_k \left(P_k^2 + \Theta_k^2 + Z_k^2 - 2U_k\right) = \sum_{k=1}^{m} n_k E_k; \tag{16}$$

$$K = \sum_{k=1}^{m} n_k \rho_k \Theta_k = \sum_{k=1}^{m} n_k K_k,$$

where the subscript k refers to the value of the given quantity for the k-th cell.

The problem now reduces to the determination of the maximum of the probability P as a function of the variables $n_1, n_2, \ldots, n_m$ which are subject to the conditions (16). Since, after finding the maximum we intend to go over to a continuous and differentiable phase density (which obeys Boltzmann's equation), we can consider the variables n_m to be continuous.

The Lagrangian for the variational problem can be taken as

$$L = \ln P + k_1 \sum_k n_k + k_2 \sum_k n_k E_k + 2k_3 \sum_k n_k K_k, \tag{17}$$

where k_1, k_2, and k_3 are Lagrange multipliers.

Differentiating with respect to n_k, we obtain

$$\frac{\partial L}{\partial n_k} = -\frac{d \ln (n_k!)}{d n_k} + \ln p_k + k_1 + k_2 E_k + 2k_3 K_k.$$

To find the meaning of the derivative of ln(n$_k$!) we will assume that the occupation number for the given cell is

$$n_k \gg 1. \tag{18}$$

Then, with sufficient accuracy, we have that

$$\frac{d \ln (n_k!)}{dn_k} = \frac{\ln [(n_k + 1)!] - \ln (n_k!)}{(n_k + 1) - n_k} = \ln(n_k+1) = \ln n_k. \tag{19}$$

Substituting this expression into the equation of $\partial L/\partial n_k$, we find after rearrangement that

$$n = p e^{k_1+k_2E+2k_3K}, \tag{20}$$

where the subscript $\underline{k}$ has been discarded, as it is no longer necessary. Instead of the occupation number of a phase cell, we now introduce the phase density f:

$$n = f d\gamma/\Gamma, \tag{21}$$

and obtain the following expression for the most probable phase density:

$$f = e^{k_1+k_2E+2k_3K}. \tag{22}$$

The condition $n_k \gg 1$ has an important basic significance. In the present problem it is determined by the necessity to change over from a discrete distribution to a continuous one and is therefore not associated with the mathematical method used by us, but with the nature of the physical problem. At the same time it has a very important consequence, namely, that all conclusions about the phase distribution are meaningful only for those values of the phase coordinates for which the occupation of the phase cells is sufficiently high. In particular, the Expression (22) for the phase density has no meaning for large values of the coordinates (outside the main body), as well as for high velocities (greater than the escape velocity).

4. Investigation of the Phase Distribution for Dynamically-Definable Galaxies

To obtain the physical meaning of the solution found, we transform the exponent in Eq. (22):

$$\begin{aligned} k_2E + 2k_3K &= k_2 (\mathrm{P}^2 + \Theta^2 + Z^2 - 2U) + 2k_3\rho\Theta = \\ &= k_2 [\mathrm{P}^2 + (\Theta - \Theta_0)^2 + Z^2 - 2U_1], \end{aligned} \tag{23}$$

where

$$\Theta_0 = -\frac{k_3}{k_2}\rho \tag{24}$$

and

$$U_1 = U + \frac{1}{2}\Theta_0^2. \tag{25}$$

Here, Θ_0 is the velocity of the centroid. We see that it is at right angles to the galactic meridian through the given point. From the hypotheses that we made at the beginning we have obtained the rotation of the galaxy about its axis of symmetry and Θ_0 gives the velocity of the centroid.

Eq. (24) shows that Θ_0 is proportional to ρ. Therefore, the angular velocity of the centroid,

$$\omega_0 = \Theta_0/\rho = -k_3/k_2 = \text{const} \tag{26}$$

is constant and, consequently, we obtain Theorem I.

Theorem 1. The most probable form of rotation for dynamically-definable galaxies is rigid-body rotation.

In our paper [I] we have shown that, judging by all the data, this in fact agrees with observations. On the other hand, from Eqs. (22) and (23) it can be seen that the dispersions of the three velocity components are the same and are equal to

$$\sigma^2 = -\frac{1}{2k_2} = \text{const.} \tag{27}$$

From this we obtain the second theorem:

Theorem II. The most probable distribution of the residual stellar velocities at all points of dynamically-definable galaxies is Maxwell's distribution with the same dispersion for all points of the system ("isothermal" distribution).

Integrating (22) over all the velocity space and taking (23) into account, we obtain the star density ν,

$$\nu = Ce^{\frac{U}{\sigma^2} + \frac{\theta_0^2}{2\sigma^2}}, \tag{28}$$

where

$$C = (2\pi)^{3/2}\sigma^3 e^{k_1} \tag{29}$$

is a constant which is determined by the condition that the integral of ν over all of the galactic volume * must be equal to N, the number of stars in it.

The physical meaning of the Lagrange multipliers has also become clear. If according to (27) k_2 determines the dispersion, k_3 on the basis of (26) determines the angular velocity of rotation, and, finally, according to (29) k_1 determines the mass, for which it is necessary to know the potential (i.e., the mass distribution), the size and shape of the galaxy, i.e., it is necessary, first of all, to solve the dynamical problem. From (28) it follows immediately that the level surfaces of the star density are given by the equation

$$U + \frac{1}{2}\omega_0^2\rho^2 = C. \tag{30}$$

This equation is the same as the analogous equation in the classical problem of the equilibrium figures of rotating liquid bodies and it will be used extensively by us in the following.

To eliminate the potential from Eq. (28), we take its logarithm and apply the Laplace operator. Then, using Poisson's Equation

$$\Delta U = -4\pi G m \nu,$$

where G is the gravitational constant and m is the average stellar mass, we obtain a differential equation for the star density:

$$\sigma^2 \Delta (\ln \nu) = -4\pi G m \nu + 2\omega_0^2. \tag{31}$$

This is the fundamental equation for the problem of determining the distribution of mass inside the D-galaxies. In expanded form it is written as:

$$\sigma^2 \left\{ \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial \ln \nu}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 \ln \nu}{\partial\theta^2} + \frac{\partial^2 \ln \nu}{\partial z^2} \right\} = -4\pi G m \nu + 2\omega_0^2. \tag{31'}$$

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* Here, as everywhere else, we have in mind the volume of the main body of the galaxy.

In the case when the star system possesses rotational symmetry, the second term in the bracket must be omitted.

Eq. (31 ') is too complicated to be worth an attempt to integrate it in the general case. However, a special case of the solution is conspicuous. In fact, in the left-hand side the star density appears only through the derivative of its logarithm. Therefore, if we restrict ourselves to the investigation of sufficiently "smooth" models, then the absolute magnitude of the left-hand side will be small. Hence, neglecting the left-hand side, we obtain the special solution:

$$2\pi G m \nu - \omega_0^2 = 0$$

or

$$\nu_0 = \frac{\omega_0^2}{2\pi G m}. \tag{32}$$

This is a very important result which leads to further conclusions. The following theorem is proved by Eq. (32).

Theorem III. In any dynamically-definable galaxy, if the star density varies smoothly enough, the star density is constant in the first approximation and is proportional to the square of the angular velocity of rotation.

Before we proceed to further deductions, we must make two remarks of a fundamental nature:

1) Firstly, if we note that the product $m\nu$ represents the density of matter $\underline{d}$:

$$m\nu = d,$$

then Eq. (32) can be rewritten in the form:

$$\frac{\omega_0^2}{2\pi G d} = 1,$$

which shows that the value of the star density obtained by us as a first approximation does not satisfy Poincare's criterion for the upper limit of the angular velocity [1]. We thus obtain Theorem IV.

Theorem IV. Dynamically-definable galaxies with quasi-constant star density[*] are dynamically unstable.

2) ν = const cannot be an exact solution of any physical problem. In fact, from Eq. (28) it follows that with ν = const, the potential must be a function of ρ only (Θ_0 is a function of ρ). But this is not physically possible, since in this case the component of the gravitational field parallel to the $\underline{z}$ axis (axis of symmetry of the galaxy) will be equal to zero, while the component of the force along the radius ρ will be independent of $\underline{z}$. Consequently, in this case we would have a uniform circular infinite cylinder, rotating about its axis. We can consider this case to be physically meaningless.

To obtain dynamical stability and a physically real model, we must find the second approximation to the solution of Eq. (31). Taking

$$\nu = \nu_0 + \nu_1, \tag{33}$$

(where ν_0 is the first approximation to ν given by (32), while ν_1 is a small quantity by comparison with ν_0) and substituting ν into (31), we can linearize the equation and obtain, after transformation, the differential equation from which the second approximation can be obtained:

$$\Delta\nu_1 + 2\,\frac{\omega_0^2}{\sigma^2}\,\nu_1 = 0. \tag{34}$$

This equation is well known in mathematical physics and is called the Helmholtz Equation (for example, see [2]). In particular, it was used by H.Poincare and J. Jeans in developing a theory of ellipsoidal figures of equilibrium.

We will study Helmholtz's Equation in greater detail below. We will only make a few brief remarks now.

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[*] That is, with a density constant in the first approximation.

For brevity, in what follows we will refer to the uniform density ν_0 as the primary density and to ν_1 as the additional density. The effect of the additional density is mainly that it determines the formation of central nuclei, which are in fact observed in most galaxies. Because of the additional density, the quantity

$$\Omega = \frac{\omega_0^2}{2\pi G m \nu} \tag{35}$$

decreases and hence the dynamical instability of the galaxy decreases.

In addition, as we will see below, the additional density leads to the formation of certain structural details in the galaxies, in particular, to the formation of an annular structure, as well as of a system of planet-like condensations similar to beads, sometimes observed in galaxies (for example, NGC 4324). The annular structure sometimes exists together with the spiral (for example, in M31).

5. The Equilibrium Figures of Galaxies

As the result of the analysis carried out in the preceding paragraphs, we found that the dynamically-definable galaxies have the following properties: 1) they rotate as a rigid body, 2) they are uniform with respect to the distribution of mass (the density is everywhere the same), and 3) they behave as continuous bodies in the sense that the methods of hydrodynamics are applicable to them. This leads to the possibility of using, in the case of galaxies, the classical theory of the equilibrium figures of rotating uniform liquid bodies.

Referring for details to the textbooks (for example, see [3, 4]), we recall that, depending on the numerical values of the quantity Ω in Eq. (35), there exist the following equilibrium ellipsoids:

Values of Ω	Type of equilibrium ellipsoids
1. From 0 to $\Omega_1 = 0.1871\ldots$	3 ellipsoids, of which two are Maclaurin ellipsoids and one is a Jacobi ellipsoid
2. From Ω_1 to $\Omega_0 = 0.2247$	2 Maclaurin ellipsoids
3. With $\Omega = \Omega_0$	1 Maclaurin ellipsoid
4. With $\Omega > \Omega_0$	No ellipsoids

In the cases when there are two Maclaurin ellipsoids ($\Omega < \Omega_0$), to each value of Ω there corresponds one "planetary" and one "disc-shaped" Maclaurin ellipsoid. The planetary ellipsoid is slightly oblate and as Ω tends to zero it tends to a sphere. The disc-shaped ellipsoid is always very oblate, it is always similar to a circular flat disc and as Ω tends to zero, it tends to an infinite thin flat layer. As Ω increases and tends to the value $\Omega_0 = 0.2247\ldots$, the oblateness of the first ellipsoid increases, while that of the second decreases, and when $\Omega = 0.2247$ the ratios of the semi-axes of both ellipsoids become the same and equal to

$$e_0 = \frac{a-c}{a} = 0.6323\ldots$$

A Jacobi ellipsoid is an elongated triaxial ellipsoid which rotates about its minor axis. It only exists for small values of $\Omega \leq \Omega_1 \leq 0.1871\ldots$. If we take that $a > b > c$, then as Ω tends to zero the semiaxes b and c tend to equality, and the axis $a \to \infty$. Thus, for small values of Ω Jacobi's ellipsoid is "needle-shaped," i.e., it has the form of a long and thin needle with pointed ends. As Ω increases and tends to $\Omega_1 = 0.1871\ldots$, the axes a and b tend to equality and at $\Omega = \Omega_1$ Jacobi's ellipsoid becomes the biaxial Maclaurin ellipsoid and its oblateness becomes equal to

$$e_1 = \frac{a-c}{a} = 0.4172\ldots$$

It is not hard to find examples of galaxies having the form of Maclaurin ellipsoids. E-galaxies and also the nuclei of S-spirals are related to the planetary ellipsoids. The presence in some of them of an equatorial rib and the outflow of matter along it can be easily fitted into the picture provided by the Roche model.

Disc-shaped galaxies also occur in adequate numbers. We can take as belonging to this type all of the varieties of S-spirals, including SO-spirals in which the spiral structure changes into a system of rings (for example, NGC 4826). Their disc-like shape is particularly apparent when these galaxies are observed edgewise (for example, NGC 4565).

Finally, if we accept the hypothesis that SB-spirals represent the result of the outflow of a stream of star gas from the ends of a primary body, then we can consider that SB-spirals are the result of the disruption of needle-shaped ellipsoids (for example, NGC 1300, NGC 7741, and others).

The fact that needle-shaped galaxies are observed most frequently in the form of SB-spirals, characterized by the escape of matter from the ends of the primary body, can be readily understood if we take into account that of all the ellipsoid types, Jacobi ellipsoids on the average correspond to the smallest values of Ω and because of this these galaxies are the least stable. A question arises, however, are there any examples of needle-shaped galaxies which are not SB-spirals? Such spirals must occur among the galaxies usually classified as "spirals seen edge-on." In fact, there is a marked difference in appearance between the various examples of this type of galaxies, as a comparison of such galaxies as, for example, NGC 4565 and NGC 4631 will show. In the former we can clearly see the central nucleus and a dark equatorial band, i.e., the features we can expect to observe in normal spirals; on the other hand, in NGC 4631 we do not see any signs of a nucleus, nor of a dark equatorial band. This gives us a reason for assuming that in the latter case we are observing a needle-shaped galaxy from one side.*

The presence of two different types of galaxies (disc- and needle-shaped) among the S-spirals must affect their observed distribution of oblateness. As can be easily shown, the fraction of galaxies with a high oblateness, usually taken to be the galaxies "seen edge-on," will be larger in the case when needle-shaped galaxies exist than if they are absent. But the excess of the type indicated has been known for a long time, as already in 1920-1922 it was discovered and investigated by Reynolds [5].

So as not to digress, we will restrict ourselves here to the above remarks and postpone a more detailed discussion of this question to another paper. Here, we have still to show that needle-shaped galaxies belong to the class of dynamically-definable galaxies.

6. On the Existence of Needle- and Pear-Shaped Galaxies

In the derivation of the most probable phase distribution in §3, we have assumed that the star system possesses rotational symmetry in its mass distribution. The integral of areas relative to the symmetry axis exists for each star only if this condition holds.

We will now show that the phase distribution obtained by us remains valid for a wider class of bodies of which the triaxial ellipsoid is a special case. That is, we will prove Theorem V.

Theorem V. The class of dynamically-definable star systems includes also any system rotating as a rigid body whose density distribution is symmetrical with respect to a plane passing through the axis of rotation and, at the same time, whose velocity distribution is symmetrical with respect to a plane which is perpendicular to the first plane.

Let $f(x, y, z; \dot{x}, \dot{y}, \dot{z})$ be the phase distribution. If we take the planes of symmetry indicated above to be the coordinate planes (x, z) and (y, z), then the symmetry property expressed in Theorem V can be written down in the form of an equality:

$$f(x, y, z; \dot{x}, \dot{y}, \dot{z}) = f(x, -y, z; \dot{x}, \dot{y}, \dot{z}) = f(x, y, z; -\dot{x}, \dot{y}, \dot{z}) = \\ = f(x, -y, z; -\dot{x}, \dot{y}, \dot{z}). \tag{36}$$

If in dividing the phase volume Γ we take the symmetrically-placed cells to be identical, then the occupation numbers of the phase cells will also have the Symmetry Property (36)

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* NGC 2188, 3034, 3556, and 2796 can serve as other examples of needle-shaped galaxies.

$$n_k = n(x_k, y_k, z_k; \dot{x}_k, \dot{y}_k, \dot{z}_k).$$

In a system of coordinates rotating with an angular velocity ω_0 the equations of motion for a free point will be

$$\ddot{x} - 2\omega_0\dot{y} - \omega_0^2 x = \frac{\partial U}{\partial x};$$

$$\ddot{y} + 2\omega_0\dot{x} - \omega_0^2 y = \frac{\partial U}{\partial y}; \quad \ddot{z} = \frac{\partial U}{\partial z}, \tag{37}$$

where U is the potential.

It is clear that U will have the same symmetry as the mass distribution, i.e.,

$$\frac{\partial U(x, y, z)}{\partial y} = -\frac{\partial U(x, -y, z)}{\partial y}.$$

Multiplying the first of the Eqs. (37) by $-y$, the second by x and adding the two, we obtain, after the simple transformations usual in such cases, the equation

$$\frac{d}{dt}(x\dot{y} - y\dot{x}) + 2\omega_0(x\dot{x} + y\dot{y}) = x\frac{\partial U}{\partial y} - y\frac{\partial U}{\partial x}. \tag{38}$$

Let us now write down the same equation for another star whose phase coordinates are $(x, -y, z; -\dot{x}, \dot{y}, \dot{z})$. We will have

$$\frac{d}{dt}(x\dot{y} - y\dot{x}) - 2\omega_0(x\dot{x} + y\dot{y}) = -\left(x\frac{\partial U}{dy} - y\frac{\partial U}{\partial x}\right).$$

Adding this equation to Eq. (38), we obtain, for a system consisting of two symmetrically-placed stars, the equation:

$$\frac{d}{dt}(x_1\dot{y}_1 - y_1\dot{x}_1) + \frac{d}{dt}(x_2\dot{y}_2 - y_2\dot{x}_2) = 0$$

or

$$(x_1\dot{y}_1 - y_1\dot{x}_1) + (x_2\dot{y}_2 - y_2\dot{x}_2) = C, \tag{39}$$

where the subscripts 1 and 2 denote the coordinates of the first and second stars. Thus, in the case of the symmetry described in Theorem V, the angular-momentum integral exists for each pair of symmetrically-placed stars and as we have agreed to take the occupation numbers of symmetrically-placed phase cells equal, then the expression for the additive parameter

$$K = \sum_{k=1}^{m} n_k \rho_k \Theta_k$$

will remain unchanged. Because of this all the other results connected with the most probable phase distribution will also remain unchanged. The only unimportant difference is that in this case we must consider the star system as formed not from individual stars, but from pairs of symmetrically-situated stars and these will be our material particles.

The applicability of the theorem formally proved by us is considerably wider than is necessary for the physical problem that we are investigating. In fact, with the most probable phase distribution, the distribution of velocities at any point is Maxwellian, i.e., it possesses spherical symmetry. Therefore, it is symmetrical with respect to any plane which passes through the center of the velocity distribution [the point (0, 0, 0)] in the rotating system of coordinates. As regards the mass distribution, the condition of symmetry with respect to a plane passing through the axis of rotation is satisfied not only by triaxial ellipsoids and similar bodies, which are symmetrical with respect to both of their principal planes passing through the minor semiaxis, but also by pear-shaped equilibrium bodies which were investigated classically by A. A. Liapunov, H. Poincare, G. Darwin, L. Lichtenstein, and others.

We have already discussed the question of the occurrence of galaxies with the shape of triaxial ellipsoids. Similarly, asymmetric pear-shaped galaxies are also often observed. We can even state that this type of assymmetry is almost always present to some extent. It is noticeable, for example, in the needle-shaped galaxy NGC 4631 which we have mentioned already. It is also noticeable in the normal-spiral galaxy seen edge-on: NGC 4565. In the most marked cases of this asymmetry it happens that S-galaxies have only one spiral arm (for example, NGC 4038).*

Thus, on the basis of the theory considered by us, the SB-spirals are the result of the rotational instability of ellipsoidal or pear-shaped bodies. Depending on the value of the star density, the angular velocity of rotation, and the presence of a central condensation (the nucleus),we have different forms of SB-spirals. In the absence of a central condensation, with a high average star density and a high angular velocity,we have a large outflow in the form of wide streams (for example, NGC 7741). In the presence of a very marked central condensation we have thin streams, which corresponds to a low value of the star density at the ends of the main body (examples: NGC 1300, 1530, and others).

Using the outflow theory, we can explain fairly simply some of the details of SB-spirals which at first sight seem to be unimportant. For example, the wide streams (as in NGC 7741) curve markedly inwards to the center because of the gradual shortening of the main body as its mass decreases. On the other hand, the thin streams curve inwards very slightly and do not depart significantly from a circular shape, because in this case the shortening of the main body takes place much more slowly.

In many SB-spirals the peculiarity is observed that the beginning of the stream does not coincide with the end of the main body. On the basis of the outflow process described above, this phenomenon can be explained in the following way. The escape of stars from the main body, naturally, mainly takes place at the expense of the stars with the highest absolute (i.e., relative to the system of coordinates with origin at the center of mass of the galaxy and with axes fixed in direction) velocities, because it is necessary for the velocity of a star to exceed the local escape velocity before it can break away. Therefore, the stream of star gas must mainly consist of stars whose absolute velocity is greater than the circular velocity of the local centroid. Because of this, the stars that have broken away will continually overtake the main body as the galaxy rotates. On the other hand, in the remaining part of the main body the circular velocity of the centroid will decrease because of the loss of the stars which overtake the centroid. Therefore, the stream as a whole will start to overtake slightly the main body and this slow advancing will also continue after the outflow has stopped.

We can often observe not one, but two or even more thin streams forming a series of rings or, more accurately, a series of sections of slowly-contracting spirals (for example, NGC 488 and NGC 1398). This can be explained by assuming that at the end of one period of outflow, the main body,because of gravitational contraction, takes the shape of another equilibrium figure. As the result of this, its moment of inertia about the axis of rotation is decreased and the angular velocity is correspondingly increased. This may initiate another process of outflow, and so on.

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* At this stage it may be useful to point out that the reason why Chandrasekhar and G.Kuzmin, who investigated the problem of finding the general solution for a stationary star system, arrived only at the solution of the special form: to systems with rotational (in our terminology) symmetry (if the solutions with screw symmetry are not considered to be physically completely unreal), is that these authors imposed certain formal mathematical conditions on their solutions.In particular,Chandrasekhar imposed the condition that the solution should give differential motion of the centroids, while Kuzmin the condition that among the integrals of motion in the gravitational field of the system there should be an integral linear in the velocities which, as can be seen from Kuzmin's analysis, is equivalent to the condition for the existence of the angular-momentum integral.

The problem of the statistical mechanics of normal spirals of the type of our galaxy will be investigated by us in another paper, in which we will show how the method of additive parameters can be used to interpret the differential(nonrigid-body)rotation observed in these galaxies. Therefore, here we will restrict ourselves to a few brief remarks on the morphology of the galaxies. If we assume that SB- and SO-spirals are formed as the result of the disruption of dynamically-unstable needle-shaped galaxies, then there can be no objections to the assumption that normal spirals are also produced in the same manner.

It is clear that these considerations must be verified by quantitative calculations on the basis of the theory of nonstationary processes, which we will not carry out here.

From the above discussion it follows that in SB-spirals the arms must be curled inwards, even if in some cases this is very slight. In this respect the results of the present theory differ from the results of the well-known theory of Lindblad. We can expect that Linblad's theory is applicable to galaxies of the type of NGC 2841, 7217, etc., which have a massive nucleus in the form of an oblate ellipsoid in which from various points along the equator (not necessarily at the opposite ends of a diameter) there is an emission of thin spiral streams as the result of the instability of circular orbits outside the main body of the galaxy.

We will investigate the problem of normal spirals in the next paper.

In conclusion, we will briefly discuss the solution of Helmholtz's Equation and the ring structure of galaxies which follows from it.

7. The Formation of Central Condensations and Annular Structures in Galaxies

Helmholtz's Equation for the additional density has the form

$$\Delta u + k^2 u = 0, \tag{40}$$

where

$$k^2 = 2\frac{\omega_0^2}{\sigma^2}.$$

Inside the main body, the unknown function u can take both positive and negative values. However, the negative values cannot be greater in absolute magnitude than the primary density ν_0, otherwise the star density would become negative. Besides, this is also unacceptable for purely formal reasons,since $|u| < \nu_0$. For the same reason, u cannot be negative outside the main body.

To solve Eq. (40) it is necessary to formulate the appropriate boundary problem, i.e., to specify the value of either u, or of its derivative along the normal to some closed surface which represents the external surface of the galaxy.

However, as an example,we will restrict ourselves to the investigation of one simple case, that of a very flat spheroid which we will take as a flat circular disc.

In expanded form in cylindrical coordinates the equation can be written in the form

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial u}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 u}{\partial\theta^2} + \frac{\partial^2 u}{\partial z^2} + k^2 u = 0, \tag{41}$$

whose solutions can be found in textbooks of mathematical physics (for example, see [2], vol. 3, 4th edition, § 154).

To separate the variables, we take

$$u = P\Theta Z, \tag{42}$$

where P, Θ and Z are functions of the variables ρ, θ, and z, respectively. For brevity we will call them the radial, azimuthal and vertical factors, respectively. Substituting (40) into (39), we obtain for P, Θ, and Z the following equations:

$$\Theta'' + p^2\Theta = 0;$$

$$Z'' - l^2 Z = 0; \tag{43}$$

$$P'' + \frac{1}{\rho}P' + \left[(k^2 + l^2) - \frac{p^2}{\rho^2}\right]P = 0, \qquad (43)$$

where p and l are new constants (p is an integer which we can consider to be always positive). Since Θ must be a periodic function with period 2π and it does not matter where we start measuring θ, in the case of the azimuthal factor we will have

$$\Theta = \sin p\theta \text{ (when } p \neq 0\text{) and } \Theta = 1 \text{ (when } p = 0\text{)}. \qquad (44)$$

For the vertical factor we obtain the formal solution

$$Z = Ae^{lz} + Be^{-lz}.$$

However, Z must obviously satisfy the condition that $Z \to 0$ as $|z| \to \infty$. In addition, we can take $Z(0) = 1$ since the value of the density for $z = 0$ will be determined by the value of the function P.

Thus in the case of the vertical factor we obtain the "barometric" equation*

$$Z = e^{-l|z|}. \qquad (45)$$

The third equation, as is well known from the theory of linear equations, has only one solution which remains finite at $z = 0$. This is the Bessel function of order p. If we take

$$k^2 + l^2 = q^2, \qquad (46)$$

then the solution will be

$$P = J_p(q\rho). \qquad (47)$$

Since the density must be equal to zero on the equator of the galaxy, we obtain for q a number of values, each of which corresponds to a root of the equations

$$J_p(\mu) = 0.$$

If we denote the i-th root of the Bessel function of order p by μ_i^p we will have

$$q_i^p = \frac{\mu_i^p}{\rho_0}, \qquad (48)$$

where ρ_0 is the equatorial radius of the galaxy.

In the general case, the solution of Helmholtz's Equation must be the sum of particular solutions of the type (42), i.e.,

$$u = A_0u_0 + A_1u_1 + \dots, \qquad (49)$$

where $A_0, A_1, \dots$ are constant coefficients.

From the theory of Bessel functions it is known that there exists an infinite number of positive roots for each value of p. In our problem the Bessel function $J_0(q\rho)$, of order $p = 0$, is of particular importance. It is the only one of all the Bessel functions which has a value at the coordinate origin different from zero: $J_0(0) = 1$. Therefore, the solution of the form (47) for every galaxy which has a central nucleus must contain a term corresponding to $p = 0$. If the other terms are absent ($A_k = 0, k \neq 0$), then the distinguishing feature of the galaxy must be an annular structure with unbroken rings, since when $p = 0$ the aximuthal factor $\Theta = 1$ = constant.

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* It is known that the "barometric" law for the decrease of star density with increasing z coordinate is observed in the case of our galaxy (for example, see [6]).

NGC 4826 is an example of an unbroken annular structure. It has at least three concentric rings surrounding the central nucleus. The elliptical shape of the nucleus and the rings must be the result of an inclination to the line of sight. Two concentric rings are quite clearly visible. The first ring is broken in two places which are diametrically opposite. If this break is real, and is not the result of the absorption of light by dark matter, then this indicates that for this galaxy the Solution (49) contains a term corresponding to $p = 1$.

The galacy NGC 4324 has an even more complicated appearance; in the rings surrounding the nucleus there appear equally-spaced bead-like condensations of approximately equal size and brightness, similar to the annular nebula in the Kant-Laplace nebular hypothesis. If we consider that the beads fill uniformly all of the ring, then there must be 16 of them and the corresponding solution must be of the order $p = 16$.

However, two difficulties arise. The first of these is that the first maximum of the Bessel function $J_{16}(x)$ occurs at $x = 18.1$, while the first few maxima of the function $J_0(x)$ occur at $x = 3.83$, 7.02, 10.17, 13.32 and 16.47. In other words, it appears as if inside the ring corresponding to $J_{16}(x)$ there should be at least five rings corresponding to the solution belonging to $p = 0$ which must undoubtedly exist in view of the presence of the nucleus. This difficulty, however, can be easily eliminated if we remember that the eigenvalue $\underline{q}$, on which the solution depends, is determined from Eq. (46) which contains, in addition to the number $\underline{k}$, depending only on the properties of the galaxy itself, the coefficient l of the "barometric" equation. The bigger l ; the faster is the decrease of density with height. For large values of l, and consequently of $\underline{q}$, the first root of the equation $J_{16}(x) = 0$ can be reached for comparatively small values of ρ. Thus,at least in principle, the first difficulty can be easily eliminated.

The situation is more complicated in the case of the second difficulty. Similarly to the ring in NGC 4826, the ring in NGC 4324 is also broken at two points on the opposite ends of a diameter. In the region of the break there are gaps also in the bead-like structure. The fact that, as in NGC 4826, these breaks occur exactly at the ends of the observed larger diameter of the ring,is very striking. This throws doubt upon the correctness of the "natural" assumption that the elliptical shape of the nucleus and the rings is caused only by the inclination of the galaxy to the line of sight. It is probable that in this case the nongravitational (for example, magnetic) forces have an important effect. This is very likely, particularly because an analogous phenomenon, i.e., a break in the ring at the ends of the larger diameter, is also observed as was pointed out by G. A. Gurzadian in some of the planetary nebulae, i.e., in objects of a completely different nature to galaxies; in planetary nebule the magnetic field undoubtedly plays an important part.*

All this shows that the nature of the annular structure of galaxies is quite complicated. We will not discuss it any further since our aim is only to investigate the principles of statistical theory, leaving to a future time its application to the structure of individual objects. We will only remark that the annular structure frequently exists together with the spiral, there being a characteristic superposition of the two structures.

8. Summary

We have seen that to the class of the simplest type of galaxies, or D-galaxies, there belong a number of varieties: 1) SB-spirals (barred), 2) SO-galaxies with an annular structure and, finally, 3) E-galaxies (elliptical). We have discussed the first two forms in greater detail. We have not considered E-galaxies because their structure and evolution can apparently be interpreted sufficiently well in terms of the well-known Roche theory of the outflow from the equatorial rim of a rotating spheroidal body (for example, see [4], vol. 3, § 54). In addition, we concluded that there exists a new type of galaxy, which we have called a needle-shaped galaxy, that has not been previously investigated. To each of the galactic types listed above there corresponds a prototype from the classical theory of equilibrium figures of gravitating rotating uniform liquid bodies.

E-galaxies correspond to Maclaurin ellipsoids of the first kind, or planetary ellipsoids. SO-galaxies correspond to Maclaurin ellipsoids of the second kind, or disc-shaped ellipsoids. Needle-shaped galaxies correspond to Jacobi ellipsoids. The same ellipsoids correspond to the main body in SB-spirals which represent the result of rotational instability. It is probable that the needle-shaped star systems are the most unstable of the galaxies, which is also connected with their flocculent structure. We did not refer to normal spirals which, as is known, are an important variety of galaxies. Our galaxy belongs to this class. The theory of normal spirals must be the subject of a separate paper. We will only point out here that in shape, of course, they resemble Maclaurin disc-shaped ellipsoids.

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* D. Evans in his article on the bright galaxies of the southern sky in Vistas in Astronomy, vol. 2 (Pergamon Press, 1956) has also pointed out this circumstance.

A few words on the meaning of the concept of quasi-stationarity in the investigation of galaxies. As we have seen, this concept allows us to apply the methods used in the study of stationary states to systems which, strictly speaking, are not in a stationary state, provided that the departure from this state takes place sufficiently slowly.

The question arises whether it is valid to apply this method in investigating galaxies, many of which, for example SB-galaxies, are in an obviously nonstationary state.

The main problem of stellar dynamics is the discovery of laws governing the evolution of star systems. Any evolutionary development of a system must be connected with a change of shape, i.e., basically with a redistribution of mass. Therefore, all evolutionary processes are basically forms of nonstationary motion of star systems.

However, it must be recognized that at present we are not able to solve in practice or even formulate the problem of nonstationary motion. At this stage, therefore, we are forced to restrict ourselves to the solution of problems of stationary motion. It can be shown that the solution of stationary problems is an appropriate and necessary stage before we pass to the solution of nonstationary problems. We have to consider the following fact. It is hard to imagine that the evolution of star systems has the same turbulent and catastrophic nature at all stages. It is more reasonable to assume that, at least in some stages, the process of evolution will slow down and during this period we can consider that in the first approximation the star system is in a state of equilibrium. States of this type are just what we have defined as quasi-stationary. In a quasi-stationary state a star system evolves so slowly that at any given instant it can attain the shape of the equilibrium figure corresponding to the dynamical conditions.

Along the line of evolutionary development of star systems, the states of quasi-stationary equilibrium must represent nodal points. Most of the various types of galaxies observed by us must correspond to these nodal points, since the higher the rate of evolution in a given phase, the smaller the number of systems of this type that would be observed. It is very likely that elliptical galaxies, SO- and normal spirals are examples of star systems in a quasi-stationary state. It is obvious, however, that even among quasi-stationary systems there are different rates of evolution. Thus, a star system which has a very flocculent structure will evolve very rapidly because of the loss of stars with velocities exceeding the escape velocity. This agrees with the fact that galaxies with a flocculent structure occur comparatively rarely. On the other hand, there are about a hundred elliptical galaxies, characterized by a very uniform mass distribution and a uniform structure, for every galaxy of another type [7].

Therefore, the investigation of quasi-stationary systems enables us to study the dynamics of the most common type of star systems. This, by itself, is enough to justify the use of the concept of quasi-stationary states as a tool in the study of the dynamics of galaxies. In addition, it enables us also to approach the problem of investigating star systems which are in phases of comparatively rapid evolution, since this phase can be considered as the process of breakdown of the preceding quasi-stationary state. Thus, for example, after the needle-shaped equilibrium figures of the galaxies have been found, the barred galaxies can be considered as the result of the emission of a stream of star gas because of the violation of Poincare's stability criterion.

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ON THE OBSERVATIONS OF THE SUN DURING THE INTERNATIONAL GEOPHYSICAL YEAR

V. A. Krat

During the International Geophysical Year observations at Soviet observatories will be mainly carried out with instruments of a standard type. The instrument which will be used in the observation and photography of chromospheric phenomena such as flares and flocculi is the chromospheric-photospheric telescope. A photoheliograph attached to this telescope will be used to study photospheric phenomena (spots, faculae). Observations are being continued with instruments already in use, namely, spectrohelioscopes and spectroheliographs. Radio observations in the meter and centimeter regions are being made at the same time.

Observations of the corona are being made as before, using Lyot coronal graphs at two mountain stations (near Kislovodsk and Alma-Ata). For the first time in the Soviet Union the intensity of magnetic fields in sunspots is being determined systematically at four observatories. At the same time, a beginning has been made with automatic recording of weak magnetic fields in active regions on the sun whose distribution over the solar disc is of major importance in forecasting solar phenomena.

The main aim of solar investigations during the International Geophysical Year is to study the effect of solar activity on the processes which take place in the earth's magnetic field and its atmosphere. This involves intensive, systematic and daily observations of phenomena which occur on the sun's surface.

Such observations cannot be carried out at a single observatory since they must form a continuous series. Only a network of special solar observatories equipped with standard instruments will be able to carry out such a program. The main instrument for observatories in the Soviet solar service is the chromospheric-photospheric telescope produced by the GOMZ factory. This instrument is a combination of two telescopes: 1) a photoheliograph used to photograph photospheric formations (mainly spots), and 2) a reflector with an interference polarizing light filter IPF, which is used to observe the sun on a wavelength corresponding to the central part of the H_α-line. The bandwidth of the filter is 0.6 A. This bandwidth may be increased to 1.2 and 2.4 A by the use of additional components. However, when this is done, the contrast of details obtained with the filter (chromospheric flares, flocculi and protuberances – filaments) is worse and hence most of the observations will be carried out with a bandwidth of 0.6 A. While the photoheliograph gives an image of the sun whose diameter is 80 mm, a second tube (essentially a chromospheric telescope), together with the IPF gives an image whose diameter is only 17 mm. This was done to facilitate the cinematography of the sun. Using a special cinecamera, photographs are taken of such fast phenomena as chromospheric flares, at intervals of between 30 secs and 2 mins. In addition, the flux of radiation from chromospheric flares and luminous flocculi in the H_α-line will be estimated in absolute units, i.e., in erg/sec, using a special photoelectric photometer in intervals between the cine-frames.

Chromospheric-photospheric telescopes have been set up and have begun working at the Crimean Astrophysical Observatory, the Mountain Station, the Main Astronomical Observatory, the Azerbaidzhan Observatory (on the Pirkuli Mountain), the Tashkent Observatory, the Alma-Ata Observatory and its high-altitude station, the

Far East Station, the Institute of Terrestial Magnetism, Ionosphere and Propagation of Radio Waves (NIZMIR, near Moscow), and the Main Astronomical Observatory of the Ukrainian Academy of Sciences. In the near future, similar telescopes will be working at the observatories of the Lvov University, the Irkutsk Station (a branch of NIZMIR) and the Kiev University Observatory. One telescope will be sent to the Chinese Peoples' Republic. A chromospheric-photospheric telescope will not be set up at the Pulkovo Observatory in spite of the exceptionally favorable conditions for solar work at Pulkovo (stable solar images), since it is not possible to observe the sun at that observatory during three months of the year (November, December and January) because of its low height above the horizon.

We thus see that the distribution of stations with latitude is such that continuous observations of solar phenomena may be carried out daily for twelve hours. This fact, together with the above, means that we now have a single international service for chromospheric flares which may be observed over the whole 24 hours in Soviet, West European and American Observatories, respectively. As is well known, chromospheric flares are of exceptional interest in geophysics since, apparently, they are the main source of streams of solar corpuscles, which cause polar aurorae and magnetic storms, as well as of high-energy particles (cosmic rays).

Old instruments such as spectrohelioscopes, which cannot be completely supplanted by chromospheric telescopes, will also be used in the observation of chromospheric flares. Spectrohelioscopes may be used to measure the widening of the emission nucleus of the H_α-line in chromospheric flares and the radial velocities of protuberances, which rise above the flares, which is very difficult to do with chromospheric-photospheric telescopes. Spectrohelioscopes are working at the Crimean Astrophysical Observatory, the Abastuman Observatory, the Tashkent Observatory, and the Kiev University Observatory. A similar spectrohelioscope is at the Czechoslovakian Observatory at Ondrzheiov, which is working in collaboration with the Soviet Solar Service.

In addition, we have a few nonstandard spectroheliographs which are being used to photograph the solar disc on a wavelength corresponding to the center of the K-line of ionized calcium. In this line luminous flocculi have the highest contrast. It is therefore necessary to have spectroheliograms in the K-line in order to see the weakest areas of faculae on the solar disc. This type of work is being carried out at the Crimean Observatory, the Mountain Station of the Main Astronomical Observatory, the Khar'kov University Observatory and, recently, at the observatory of Moscow University on Lenin Hills.

All the observatories which have major solar instruments, and which work in favorable (favorable astroclimate) conditions, have been given the important task of determining the intensity of solar magnetic fields. This involves, above all, the determination of the intensity of magnetic field of sunspots, which is very important to a complete understanding of the evolution of spots and their groups. This work is being carried out at the Crimean Astrophysical Observatory in contact with the Potsdam Observatory (German Democratic Republic), and also at the Observatory of the Institute of Terrestial Magnetism, Ionosphere and Propagation of Radio Waves, where tower telescopes are being used, and the Main Astronomical Observatory (at Pulkovo) and its Mountain Station, where telescopes of the horizontal type are employed. Recently, after the experimental work carried out in America by H.W. Babcock and H.D. Babcock, it became clear that it is important to construct charts of solar surface showing values of the intensity of weak local magnetic fields on the sun. The work of the Babcocks has shown that a general magnetic field does not exist on the sun in the form of a dipole field, and that, in fact, the field consists of a great number of low local magnetic fields with maximum intensities of a few gauss. Changes in the configuration of these fields, the appearance of fields of the order of 100 gauss or more, often accompany chromospheric flares and are associated with deep changes in active regions of the sun. The work of astronomers at the Crimean Astrophysical Observatory has led to the same conclusion. Automatic recording of the intensity of weak magnetic fields (of the order of one or a few gauss) and the construction of charts showing the distribution of magnetic fields on the sun are being carried out at the above-mentioned three Soviet observatories.

During the International Geophysical Year much importance is being attached to systematic observations of the solar corona, which should be traversed by the streams of corpuscles ejected by the sun. At the same time, one may try to determine the intensity, direction, and velocity of these corpuscular streams and thus establish the possibility of accurate prognosis of magnetic storms. Observations of the corona in the light of its brightest lines, namely, the green, red and yellow lines, are being carried out at the Mountain Station of the Main Astronomical Observatory and the Far East Station of the Alma-Ata Observatory. Systematic observations of solar radio emission in the wavelength range 0.5-1.5 m, which is associated with the radio emission of the corona, are particularly important to the study of the motion of corpuscular streams in the corona. Such systematic observations are being carried out at the Crimean Astrophysical Observatory and also at the Radio-physical Institute of the Gor'ki University, the Mountain Station of the Main Astronomical Observatory, and at the observatory at Ondrzheiov in Czechoslovakia in collaboration with the former observatories. Observations in the centimeter range (wavelengths of about 3 cm) are being carried out at Pulkovo. Studies of the polarization of radio emission on these wavelengths may give interesting information on the intensity of magnetic fields in the densest regions of the corona (coronal condensations) and even in the chromosphere.

Observations of solar radio emission which have been begun in the Soviet Union are only first steps in this direction. In the near future radio observations of the sun will be begun at many stations of the Solar Service with the aim of providing a continuous solar radio emission service which would function daily over at least 12-14 hours.

Rapid processing of results and their timely publication have become particularly important. Urgent publication of the results obtained by the Soviet Solar Service will be available in the "Solar Data" bulletin which is published by the Pulkovo Observatory on behalf of the Commission for Solar Studies. In addition to the data published up to now (position of spots, facular areas, isophotes of the corona, position of filaments and protuberances, chromospheric flares), the intensity of magnetic fields will be given on daily charts of the solar surface given in this journal.

Final summaries of all this work will, as before, be given in the "Catalogues of Solar Activity."

The main organizational center which directs the transmission of urgent data, and which acts as a link with the International Center for the International Geophysical Year, is the Institute of Terrestial Magnetism, Ionosphere and Propagation of Radio Waves.

Main Astronomical Observatory of the
USSR Academy of Sciences

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BRIEF COMMUNICATIONS

METEORITIC MATTER AT THE PLACE OF FALL OF THE TUNGUS METEORITE

A. A. Iavnel'

Results of an investigation of the magnetic fraction of samples of soil taken in 1927-1930 in the neighborhood of the place of fall of the Tungus meteorite are reported. Under a microscope two sorts of small metallic particles and magnetite (?) globules were found in the soil. Spectral analysis has shown that the particles are composed of nickel-iron. It is concluded that this substance is part of the Tungus meteorite which, judging by the form and the dimensions of the nickel-iron particles, was of the iron type.

It is now 50 years since the fall of the Tungus meteorite in the Siberian taiga on June 30, 1908. The phenomena which accompanied this exceptionally large-scale fall have been studied by many authors during the last 35 years. The most complete summary of data on the Tungus meteorite can be found in the monograph by E. L. Krinova, which was published in 1949 [1].

However, in all these papers the nature of the substance of which the Tungus meteorite was composed remains an open question. This is explained by the fact that so far no traces of meteoritic material have been found at the place of fall. Various suggestions have been put forward by a number of authors to explain this fact.

L. A. Kulik [2] gave a summary of the studies carried out by various expeditions to the place of fall of the Tungus meteorite and considered that several meteoritic masses fell in that region, and that their masses were up to hundreds of tons, so that they penetrated the earth to a depth of 25-50 m, and hence were not found.

Recently, K. P. Staniukovich and V. V. Fedynskii [3, 4] have published important theoretical papers on the destructive effect of meteor impacts, and showed that large and gigantic meteorites which fall on the earth's surface with residual cosmic velocity (greater than 4-5 km/sec) produce a powerful explosion, as a result of which the meteoritic matter is either partly or fully evaporated. It follows that under these conditions one is unlikely to find any large pieces of meteoritic matter.

On the basis of these ideas, E. L. Krinov [1] concluded that in the case of the Tungus meteorite one can, at most, find only relatively small fragments of the meteorite, scattered in the neighborhood of the place of fall. In addition, he did not exclude the possibility of finding small individual samples if the meteorite fell in the form of rain. The absence of any finds of meteorites is explained by E. L. Krinov by the fact that during Kulik's expedition no search was made for meteoritic matter in the neighborhood of the place of fall. These conclusions open new possibilities in the search for meteoritic matter due to the Tungus meteorite.

It is natural to expect that during the explosion part of the meteoritic matter settled down on the earth's surface in the form of dust. It is therefore of major interest to carry out studies on samples of soil taken from the neighborhood of the place of fall of the Tungus meteorite, where one is clearly more likely to find traces of meteoritic matter. For this reason, in the program of a new expedition which it was recently planned to send to the place of fall of the meteorite, provision was made for taking samples of soil, although it is difficult to predict in advance to what extent remains of the meteorite have been preserved during the 50 years since the fall.

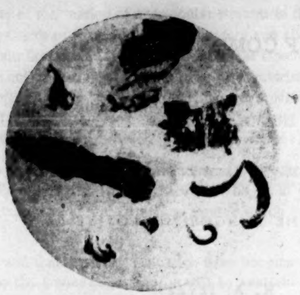

Fig. 1. Nickel-iron particles. × 35.

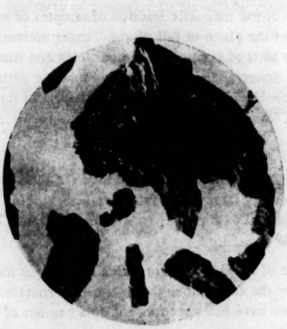

Fig. 2. Nickel-iron oxidized particles. × 35.

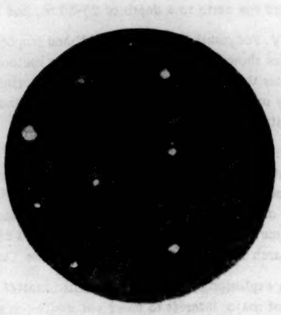

Fig. 3. Magnetite (?) particles. × 120.

Geological samples taken from the neighborhood of the place of fall of the Tungus meteorite by L. A. Kulik during the 1927-1930 expedition, i.e., 20 years after the fall of the meteorite, are still in the possession of the Committee for Meteorites of the AN SSSR. On the suggestion put forward by Academician V. G. Fesenkov, the present author has carried out an investigation of these samples. Altogether there were 89 surface and other samples of soil and specimens of rocks. From among these specimens 13 most characteristic ones were chosen (42-541 g in weight), of which 6 were taken from surface layers of soil in sections, pickets, pit-bogs, etc., and 7 borings from bore No. 1 taken at different depths (up to 23 m). These samples were taken in the region of the South Marshes, which is assumed to be the place of fall of the meteorite.

Each sample was ground down in an agate mortar and the magnetic fraction was removed from it by means of a magnet and was examined under a microscope under ×30 magnification.

It was found by visual inspection that the meteoritic fraction consists mainly of black magnetite crystals which form a part of rocks. In addition, small particles of the following three types were found:

1) Metallic particles, silvery white in appearance and in the form of thin corrugated "petals" and "tendrils," similar to shavings, and whose dimensions are of the order of tenths of a millimeter (Fig. 1).

2) Oxidized metallic particles, mainly planar and bounded by lines forming acute angles, or particles in the form of bars with maximum linear dimensions of a few millimeters and having slightly fused surfaces and edges (Fig. 2).

3) Black shiny spheres, apparently of magnetite, and having a diameter of 30-60 microns, among which one sphere silvery white in appearance may be seen (Fig. 3).

Particles of each type were separated from the magnetite as completely as possible and then, together with the remaining part of the magnetic fraction (except for the spheres), subjected to spectral analysis. The specimens were fired in an activated ac arc between carbon electrodes, and their spectra were photographed on a quartz spectrograph of average dispersion.

This analysis showed that the metallic shavings and oxides consist of nickel-iron. Quantitative spectral analysis, which included photometering of the spectra of the specimens and the corresponding standards, showed that the shavings contain about 7% of nickel and 0.7% of cobalt, while the oxides contain 4% of nickel and 0.3% of cobalt; traces of Si, Mg, Mn, Al are very small indeed and may be due to contamination by silicates, and elements such as Cr, V and W were not found at all.

The nickel content of magnetite is of the order of tenths of a percent or less, and, as was established, depends on the quantity of very small metallic particles which could not be removed from the magnetite.

The obtained data indicate that the metallic particles have a composition which is typical of meteoritic iron and does not resemble the composition of artificial iron alloys. The latter fact excludes the possibility of contamination of the samples by particles of the instrument. The low nickel and cobalt content of the oxidized particles may be explained by the well-known phenomenon which involves the loss of these elements when meteoritic iron becomes oxidized.

A comparison of our data with results of an investigation of the place of fall of the Sikhote-Alinskii meteorite, which was carried out by E. L. Krinov and S. S. Fonton [5], shows good agreement between them. Near the place of fall of the latter meteorite, metallic particles in the form of shavings and, apparently, magnetite spheres, were also found. One must note that the spheres which we found were discovered in borings carried out over an area of about 36 cm^2. Remembering that they may penetrate to a depth of 18 m, which is large compared with the length of the samples in which they were discovered, one may assume that the relative density of the spheres per unit area of the surface is quite large.

In the recent paper by Rinehart [6], an account is given of an investigation of the soil in the region of the Arizona crater, which led to the discovery of fine fragments of nickel-iron and iron oxides of meteoric origin.

Thus, both in their chemical composition and by their form, the particles which have been found by us are analogous to those which have been found at the places of fall of major iron meteorites. These particles are evidently due to the disintegration of the meteorite during the explosion. As far as the spheres are concerned, the work of E. L. Krinov on the Sikhote-Alinskii meteorite has shown that drops of matter are blown off the molten surface of the meteoric body which moves through the terrestial atmosphere with a cosmic velocity, and then settles on the earth's surface in the form of small spheres. Obviously, in this case the spheres are a part of the dust trail formed by the falling meteorite. Undoubtedly, metallic fragments of this type could not have been formed during the disintegration of stone meteorites containing small iron impurities.

It may therefore be assumed that in our case we deal with matter belonging to the Tungus meteorite which, according to all the data, should have been a huge iron mass. This is supported by the fact that in 1908 a considerable increase in the turbidity of the atmosphere occurred and this, according to V. G. Fesenkov, was due to the fall of the Tungus meteorite. In conclusion one must point out that investigations of samples of soil taken at the place of fall of the Tungus meteorite must be continued with new material which may be obtained by sending a special expedition to the place of fall. Results obtained so far indicate that such an expedition would be successful.

Committee for Meteorites
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CHRONICLE

THIRD SYMPOSIUM ON COSMICAL GAS DYNAMICS

Much attention is being devoted at the present time to cosmical gas dynamics, which is a relatively new branch of science contiguous with the usual gas dynamics, electrodynamics and astrophysics. The interest of astronomers in this science is explained by the fact that gas motion and the formation and scattering of gas clouds, the expansion of gas nebulae and a number of other phenomena are apparently related to the formation and development of stars. Another fact which is significant in this connection is that gas motion may be observed by a number of different methods, namely, interstellar absorption lines, shift of emission lines, expansion of shells and, finally, through observations using the λ 21 cm radio line, so that theoretical results may be compared with experimental data.

To physicists,the motion of interstellar gas is of interest mainly from the point of view of the problem of acceleration of cosmic rays. Furthermore, gas motion is linked with the magnetic field and apparently tends to increase this field. The presence of the field in its turn leads to the emission of radio waves by relativistic electrons, which considerably reduces the number of electrons reaching the earth as part of the primary cosmic rays.

For hydrodynamicists,interstellar gas is a kind of "laboratory" where its motion may be studied under very specific conditions, e.g., high effective conductivity, increased contribution of electromagnetic forces, etc.

Since 1949 symposia have been held at four-year intervals on cosmical gas dynamics and astrophysicists and hydrodynamicists have taken part in them. The third of these symposia took place between June 24-29 in Cambridge U.S.A.) A Soviet delegation took part in this symposium and among members of the delegation were V. A. Ambartsumian, L. I. Sedov, A. G. Masevich and S. B. Pikel'ner.

The delegation flew to Stockholm on June 22 in a Scandinavian Airways Company plane and hence on to New York on the same day. In the early morning of the following day (Sunday) we were met at the International Airport by the Deputy Director of the Smithsonian Institute, Dr. G. F. Schilling and Dr. L. Jacchia. After the completion of formalities, we were driven to LaGuardia Airport,from which we flew to Boston. Cambridge is a small town near Boston and has quiet green streets whose sides are continuously lined with cars. We were put up at the Continental Hotel near the Institute where the symposium took place. The sessions usually took place between 9 and 6 with a break in the middle.

The first session was opened by the Director of the Smithsonian Astrophysical Observatory, Prof. F. L. Whipple, who greeted the participants and wished them success in their work. Prof. J. M. Burgers, who was the chairman of the Organization Committee for the Symposium, gave a short introductory talk. He pointed out the advantages of collaboration between astrophysicists and hydrodynamicists and reminded us of the main problems which were considered during previous symposia. This was followed by two comprehensive review papers giving a resume of results of observational (H. C. van de Hulst) and of theoretical (E. N. Parker) work; van de Hulst (Holland) compared the accuracy of determinations of position and velocity of celestial objects obtained by different methods,such as those using interstellar emission and absorption lines, the radio line at 21 cm, direct photography, and the emission of dark clouds. He reviewed studies which have been carried out on the spiral arms, using the 21 cm line: the thickness of these arms is about 200 parsec, the mean concentration is $n \sim 1\ \mathrm{cm}^{-3}$. The average value for the whole plane of the galaxy is $n \sim 0.7\ \mathrm{cm}^{-3}$. He reported new data on 21 cm for high galactic latitudes between −50° and +60°. The main maximum of the line has a width of up to 80 km/sec. In addition, there are weak long wings usually on the shorter wavelength side and extending up to about 200 km/sec. Near the center of the galaxy, motion has been observed with a dispersion of radial velocities of about 50 km/sec. It has recently been explained that these velocities are, in the main, associated with the expansion of gas away from the central region while random velocities in the center are small and are of the order of 10 km/sec. In the radio spectrum of the source Sag A, which is located in the direction towards the center of the galaxy, an absorption line is observed at 21 cm showing an approach velocity of 50 km/sec. Recession lines corresponding to the far side of the shell expanding away from the center are not observed. This means that the source is located inside the expanding shell and close to the center of the galaxy.

Sections through the Andromeda Nebula obtained on 21 cm have made it possible to determine the density distribution of hydrogen and the law of rotation. The mean density in the plane of the galaxy is not much less than 0.1 atoms per 1 cm^3 while the radio emission is much more drawn out than the optical emission and forms separate maxima corresponding to the arms. The linear velocity of rotation at first increases with distance from the center, reaches a maximum at $r = 5$ kparsec, and then slowly falls off.

The dispersion of velocities of clouds has, according to radio data, turned out to be about the same as that obtained from absorption lines and is about 8 km/sec. Internal motion in clouds has, according to measurements with a large telescope, a mean dispersion of 2.3 km/sec, while the form of separate components of optical lines gives a figure of 3.5 km/sec. These velocities have a largely macroscopic character. Certain conclusions may be drawn from a discussion of the structure of nebulae, particularly on the boundary between luminous and dark regions.

The paper by Parker (U.S.A.) was concerned with general principles and equations of magnetohydrodynamics and some of their applications to interstellar gas. It follows from the dispersion of the planes of polarization of stellar light that the magnetic energy density in arms ($4 \cdot 10^{-12}$ erg/cm^3) is greater than the mean kinetic energy density for clouds ($0.8 \cdot 10^{-12}$ erg/cm^3). Under these conditions,part of the energy of motion is taken up by transverse magnetohydrodynamic waves. This is due to the fact that the differential rotation of the galaxy does not produce a highly developed turbulence in interstellar gas,while magnetohydrodynamic waves in a strong field are linear and do not transmit energy to small scale motion.

From collisions of clouds,the time for the dissipation of kinetic energy has been calculated and found to e $7 \cdot 10^6$ years, which gives a mean dissipation of 10^{-27} erg/cm^{3}sec. Among the various mechanisms which ay support the motion of gas clouds, namely, gravitational instability, new stars, differential rotation of the galaxy and the formation of hot stars, only the latter can be effective. The expansion energy of shells of new stars should be almost entirely converted into radiation in accordance with the law of conservation of momentum. An estimate was made of the conditions necessary for gravitational instability. The critical dimensions for an interstellar medium are about 100 parsec while the critical density for a typical HI cloud is about 100 atoms per 1 cm^3. It is therefore concluded that interstellar medium should be in a state which is intermediate between a uniform distribution and separate self-gravitating clouds. The gas equilibrium depends to a very large extent upon the presence of a magnetic field. If a complex field is not present within the cloud, the cloud will spread out in at least one dimension.*

In conclusion, a discussion was given of the fact that the energy density of cosmic rays, the kinetic energy density of the gas and the magnetic energy density are all roughly equal. Apparently it is difficult to explain this by the acceleration of particles in the interstellar field.

The evening session opened with a paper by G. de Vaucouleurs (Australia), who reported radio observations which have been carried out on 21 cm by F. Kerr and his group in Sydney. Observations were carried out, using the radio telescope with an 11 meter mirror, which was fixed during observations, and a 4-channel receiver with a bandwidth of 40 kc (8 km/sec).

A general picture of the spiral structure of the galaxy in the southern hemisphere has been obtained and a chart has been constructed showing regions of maximum hydrogen density in the plane of the galaxy. In the direction towards the center, the gas layer has a thickness (half-density layer) of about 250 parsec, while the sun lies very close to the central plane. In the outer parts of the galaxy there are anomalies; for example, in the direction $l = 40°$ the dense part of the layer is 300 parsec above the plane of the galaxy, and in the direction $l = 240$-$250°$ (i.e., in the direction of the Magellanic Clouds) they are 500 parsec south of the plane of the galaxy. This effect cannot be explained by a simple tidal effect due to the Magellanic Clouds.

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* The existence of polarization of interstellar light shows that the field in clouds cannot be very complex. On the other hand, G. A. Shain has shown that nebulae tend to spread along the magnetic field of arms .

A smoothed picture of the distribution of hydrogen in the plane of the galaxy shows that its density is approximately constant beyond 4 to 5 kparsec from the center, but falls off rapidly in the inner regions. A comparison with a distribution of total mass (including stars) as calculated by M. Schmidt shows that the relative mass of gas rapidly increases in the outward direction reaching 15% near the sun ($r = 8$ kparsec) and 50% at $r = 13$ kparsec. Integration over the whole galaxy including stars of spherical subsystems gives 2% for the relative hydrogen content if no account is taken of the galactic corona.

G. de Vaucouleurs also reported on this work on the structure of galaxies and their rotation. He divides galaxies into four main classes: ellipticals E, lenticulars S_0, spirals S, and irregulars I. The lenticular and spiral galaxies are subdivided into two groups, namely, the ordinary, SA, and the barred, SB, with intermediate objects SAB. According to their form the lens-shaped and spiral galaxies are divided into rings (r) and spirals (s) with intermediate objects (rs). Subdivisions have been noted along spiral sequences: early (a), intermediate (b), late and very late (c and d) and Magellanic M which are transition states to irregular IA_m, IB_m and I_m. In the sequence of lens-shaped galaxies there are three stages between elliptical and spiral: early SO , mean SO^o and late SO^+. The outer ringlike structure is often observed close to the transition stage between lens-shaped and spiral galaxies (SOa).

The above morphological sequences are in good agreement with sequences associated with changes in other parameters, e.g., the mass-luminosity ratio, the ratio of the mass of gas to the total mass, polarization, etc.

The tilt of nebulae whose planes are at a small angle to the line of sight was determined from photographs obtained with a 200-inch reflector from the position of the dark lane along the edge of the galaxy. In the case of a large tilt, its sign was determined from the absorption near the ends of the bar, from the relative position of dark and bright areas in an arm, etc. A comparison of tilt with radial velocity showed that in all cases the spirals are "trailing."

R. D. Davies (England) reported on results obtained by him, using the 21 cm radial telescope at Jodrell Bank. The radio temperature of neutral hydrogen in some directions in the Cygnus and Auriga constellations, where a large number of dust clouds is present, has turned out to be much lower than in the surrounding areas (from 25 to 75°K instead of 125°). The author explained this by the cooling effect due to molecular hydrogen and metal atoms which are abundant in dense clouds. The dispersion of velocities for HI clouds in the above directions reaches up to ±10 km/sec. Separate clouds whose radio lines are observed in absorption in the spectra of sources in Cassiopeia and Sagittarius have a dispersion of internal velocities between ±2 and ±10 km/sec. These results are in agreement with the data obtained for interstellar Ca^+ and Na lines and indicate the presence of fast turbulent motion within the clouds. 21-cm observations of Cygnus-X have shown that this line is absent within the limits of the source. The magnitude of the shift in the spectrum for the part of the profile where HI emission is absent has shown that the source is located in the second spiral arm (about 6 kparsec). The radio spectrum from this source is similar to that from an HII region. The distribution of hot stars supports the suggestion that Cygnus-X is a large HII region which penetrates into the second spiral arm.

The problem of the nature of the spherical gas corona of the galaxy was discussed in the paper by S. B. Pikel'ner and I. S. Shklovskii (USSR). The intensity of the magnetic field at large distances from the galactic plane was estimated from the distribution of the density of radio emission. From the equilibrium conditions for a gas layer above $Z = 10$ kparsec under the action of magnetic pressure, cosmic ray pressure, kinetic pressure and the forces due to gravitation, the gas density was estimated to be about $0.6 \cdot 10^{-2}$ atoms per 1 cm^3. Arguments were given in favor of the suggestion that the magnetic field has a complex character and is considerably amplified by random motion of matter. The velocity of this motion is estimated to be 100 km/sec, approximately. The problem of why supersonic motion is not rapidly attenuated and thus converted into heat was discussed. The dissipation was calculated, taking into account the magnetic field, and it was shown that if the energy of the field is equal to the kinetic energy, then the dissipation decreases by a factor of 20-30. The ionization of hydrogen and the heating of gas under the action of shock waves were also calculated. Observations of the 21 cm line in Magellanic Clouds, and particularly of the Coma Cluster, indicates the presence of fast macroscopic gas motion and supports the hypothesis that the dissipation in weak magnetohydrodynamic shock waves is low (low gas temperature, leaving hydrogen unionized).

The source of the kinetic energy appears to be the nucleus of the galaxy in which rapid gas motion is observed, while nonthermal emission is concentrated towards the center, both in our own galaxy and in radio galaxies. In the discussion after the paper, D. Heeschen (U. S. A.) announced that he observed emission on 21 cm in three galaxies, namely, M 32, M 51 and M 81, while N. Dieter observed this radiation in M 33. The galaxies (except for the elliptical M 32) are surrounded by a very extended (up to 40 kparsec) and rarefied ($n < 10^{-2} cm^{-3}$) hydrogen cloud. Its total mass in spiral galaxies is greater than $10^9 M_\odot$, the velocity dispersion being about 50 km/sec, and it is possible that more distant weak wings are also present. According to Heeschen, these data confirm the existence of a halo consisting of rarefied and relatively cool gas with a large dispersion of velocities.

Van de Hulst showed that in the radio emission of the Andromeda nebula, within the limits of accuracy (0°.3), the wings of the 21 cm line are absent, and these would correspond to large velocities. On the other hand, weak wings of the lines in the emission of our galaxy at high latitudes may possibly indicate the presence of fast motion of rarefied and weakly ionized hydrogen. In any case, with the present stage of development of radio engineering, the problem of the presence of neutral hydrogen in the corona may be solved in 1-2 years.

The validity of the cloud model of interstellar gas was discussed in the paper by B. Donn (U.S.A.). The speaker showed earlier that observational data may be better explained if interstellar gas is looked upon as a single medium with density fluctuations and not as separate clouds. Various aspects of this hypothesis were also discussed in this paper, including the difficulties involved, the main of which is the low velocity in strong components of interstellar lines. It was concluded that the whole problem of the structure of interstellar gas should be examined again.

The morning session of June 25 was concerned mainly with the dissipation of kinetic energy. H. E. Petschek (U.S.A.) considered the dissipation in shock waves taking into account weak magnetic fields. The basic idea was to take into account the fact that the interaction distance between particles is equal, not to the mean free path, but to the radius of a spiral which, even with weak magnetic fields, is small. The Navier-Stokes equations do not apply to this case. Instead, a kinetic equation is written down which takes into account the field and makes it possible to carry out a qualitative analysis. One of the results is the decrease in the thickness of the front of the wave, which can be considerably less than the mean free path. This makes it possible to obtain better agreement between observations and Gold's hypothesis according to which magnetic storms with nonsudden commencement are associated with the passage through interstellar space of a shock wave produced by a current ejected from the sun. The structure of the wave is modified when the magnetic field is taken into account. An attempt was made to verify these theoretical results by laboratory experiments with shock tubes. However, the results were not very definite since the luminous region is wider than the pulse width.

Another mechanism for the dissipation of energy was discussed by L. Biermann and A. Schluter (West Germany). Magnetohydrodynamic motion in a weakly ionized gas in an HI region leads to ambipolar diffusion, i.e., systematic relative motion of ions and neutral atoms upon which electromagnetic forces do not act. Collisions lead to the dissipation of the energy of motion which is converted into heat and this dissipation is estimated to be 10^{-3}-10^{-2} erg/g sec, i.e., 10^{-21}-10^{-26} erg/cm^{3}sec. The motion is damped out in a time of the order of $2 \cdot 10^{14}$-10^{15} sec.

Collisions between ionized gas clouds were discussed in a paper by F. D. Kahn (England). Such a collision produces a shock wave which heats the clouds at the expense of kinetic energy. The interpenetration distance increases with increased relative velocity. The result may also be applied in the case of a collision between two fast nonionized clouds, since at large velocity ionization of hydrogen will occur more rapidly than simple momentum transfer. The radio source Cygnus-A was considered as an example of such a collision.

Problems associated with the formation of condensations of interstellar gas were discussed during the evening session. E. Schatzman (France) calculated the cooling of a HI region due to the excitation of low levels in C^+, Si^+, Fe^+ at different temperatures of the clouds in the range 18-1000°K. (M. J. Seaton's model). An estimate was made of the temperature difference between neutral and electron gas for which the relaxation time is $\sim 10^7$ years and which depends strongly on electron concentration. Within large clouds where the dust absorbs ultraviolet radiation which ionizes carbon, C I regions may be formed (usually in interstellar space carbon is in the CII state). CI has many low lying and easily excitable levels which have a pronounced cooling effect on the gas. Within a cloud having a temperature of about 1000°, rapid contraction of the cool central CI region may occur under the action of the pressure due to the surrounding gas. Recombination of electrons which are present in a HI region takes place at considerably slower rate than excitations, and hence electrons will continue to cool the gas, and the contraction of the central part will be prolonged. Thus, according to this author, dense gas regions may occur.

M. J. Seaton (England) reported the presence of considerable condensations of gas in planetary nebulae which were discovered during spectral studies. The density was obtained from the 3729/3726 (OII) and 7320/ / 7330 (NII) ratios. The results obtained differ among themselves. To satisfy both these ratios [and the ratio (OII)/(NII)] one may, for example, assume that in NGC 7027, $n_e = 6 \cdot 10^3$ in 99% of its volume and $9 \cdot 10^4$ in the remaining 1%. This would be in agreement with measurements of surface brightness in hydrogen lines. In other nebulae which have been studied, apart from the very uniform IC 418, there should also be appreciable density fluctuations in a small part of the volume.

The mechanism suggested by J. H. Oort, according to which stars are formed from diffuse matter, was discussed in the light of some observational data in the paper by V. A. Ambartsumian (USSR). According to Oort's mechanism, stars should be formed mainly on the periphery of an expanding region of ionized hydrogen. At the same time, observations show that the earliest and brightest giants which form short systems of the Trapezium type are usually located at the center of such an expanding region. This major difficulty associated with the above hypothesis supports, in the opinion of this author, the hypothesis that stars are formed simultaneously with the nebula, or somewhat later, mainly in the center of the nebula and possibly from bodies considerably denser than the nebula. In any case, one cannot assume that the hypothesis according to which stars are formed from dark matter compressed by ionized gas satisfactorily explains all the observational data.

In the discussion after this paper C. Wade (U.S.A.) reported studies of the 21 cm emission in the neighborhood of the almost spherical HII region which surrounds λ' Orion. The HII region is included in a thick and relatively dense shell of neutral hydrogen which is expanding with a velocity of about 8 km/sec. The shell is about 28 parsec thick and its total mass is about 45,000 $M_\odot$. M. P. Savedoff reported some calculations on the process of expansion of the HII region surrounded by neutral hydrogen. He took into account the presence of an ionization front moving relative to the gas and calculated the degree of contraction and the change in the rate of emission with time. He applied his calculations to the shell surrounding λ' Orion and the so-called Orion Loop. In this connection K. Menon (U.S.A.) reported observations of the Orion Loop on 21 cm which show the expansion of the neutral hydrogen which surrounds it. M. P. Savedoff (U.S.A.) next considered the dynamics of intergalactic collisions. The difficulty in explaining the phenomenon is the distribution of radio emission in the Cygnus-A source while optically the two galaxies are face to face and the radio emission forms two sources on either side of the galaxies. In this connection the hypothesis due to I. S. Shklovskii was discussed, according to which colliding galaxies are surrounded by gaseous coronas which came into contact with them earlier and then moved away, while the stars and the dense gas continued to come closer together.

In addition to the papers given by the participants in the conference, written contributions were received from V. G. Fesenkov, V. S. Safronov, S. A. Kaplan and D. A. Rozhkovskii (USSR).

V. G. Fesenkov's paper discusses stellar changes which can be seen on photographs of parts of the sky which are rich in stars. Artificial "stellar charts" with random distribution of points, and calculations of the probability of accidental formation have shown that tight chains with stars roughly equal in magnitude and color are apparently not accidental. The connection between chains and dark filaments and nebulae, which was also noted by D. Ia. Martynov, supports this conclusion. If these chains are not in fact accidental but physically real formations, then because of their instability their age is not very great. An estimate was made of the density of dust in thin dark filaments and this turned out to be high, namely, up to 10^{-22} g/cm^3.

V. S. Safronov reported calculations on the criterion of convective instability for a protoplanetary cloud and showed that convection cannot take place for any admissible values of temperature and density gradient. Hence convection cannot be the reason for the appearance of turbulence. The transport of matter and angular momentum due to the presence of turbulence was discussed. In distinction to C. F. von Weizsacker, V. S. Safronov considers that turbulent elements transport angular momentum, so that the inner part of the cloud will not be retarded in spite of the large angular velocity.

S. A. Kaplan studied shock waves in interstellar gas, taking into account the important role played by radiation which determines the temperature of the gas. In this case shock waves may increase the density of non-ionized hydrogen by a factor of a few tens. The method of "automodel" motion was used to study the propagation of ionization discontinuities. In particular, it was shown that a shock wave in front of such a discontinuity may increase the density in an HI zone by a factor of a few hundreds. Such increases of density may be associated with interstellar clouds.

D. A. Rozhkovskii has carried out a statistical study of globules projected on stellar clouds. The random distribution of stars is determined from Laplace's formula. The distribution of stars when a large number of identical globules is present among them, each of which has a given absorption, has been calculated. A comparison of theoretical distributions with observations (star counts over small regions) has been carried out for three areas in uniform stellar fields neighboring dark clouds. In not very dense stellar fields, small statistical deviations have been found due, apparently, to the presence of globules having diameters of 1'-2' and an absorption less than 1^m. The number of the globules should be of the order of 10^3 per square degree. In dense clouds, only isolated large globules are observed, although under the prevailing conditions one should be able to see globules with diameters 1' or less.

The session on June 27 was opened with a paper by L. I. Sedov (USSR), who gave an account of the results of his theory of nonstationary gas motion, taking into account gravitational forces in the presence of shock waves, detonation waves and the magnetic field. Models of motion of various types of outburst, detonation, and of nonstable equilibrium were considered in connection with stellar explosions. In addition, accurate solutions of the problem of motion of gas columns with a "frozen-in" magnetic field were given. Some problems on the motion of magnetized bodies in a conducting medium were formulated and solved.

G. C. McVittie (U.S.A.) reported accurate solutions of the equations of magnetohydrodynamics of compressible gas with an infinite conductivity, taking into account gravitation and magnetic fields. Two cases were considered: in the first case the potential of the self-gravitating forces have a certain functional form, and in the second, the distribution of velocities have a functional form. As an example, a discussion was given of the problem of a moving plane self-gravitating thin gas layer which moves in the direction perpendicular to itself with a constant acceleration (idealized thin clouds) and the problem of radial motion of a self-gravitating cylinder with a field directed along the axis (model of a spiral arm). The equations which have been obtained can be solved numerically and conditions have been found under which small vibrations are harmonic.

L. Mestel (England) considered a nonrotating protostar contracting in a magnetic field which penetrates also into the surrounding medium. The dust density was assumed sufficient to absorb galactic radiation in an HI region with a temperature of about 100°K. The critical radius $r_c = 10^{16}\ M/M_\odot$ up to which the flow of gas remains subsonic has been calculated.

The paper by E. Bullard (England) was concerned with the effect of convection within a body on the internal and external magnetic field. If the conductivity is infinitely high, then the field cannot penetrate in the outward direction, in accordance with Cowling's theorem. If the conductivity is finite, then the increase in the field due to internal motions has an effect (by induction) upon the external field. In this way the earth's field may change by 1% in a century. At the same time, observations indicate greater (up to 30%) changes in the earth's magnetic field. Rising currents transport part of a strong toroidal field which is assumed to exist near the nucleus and form field loops which move out from the nucleus. A simplified model has been investigated and the results may be applied also to the problem of formation of the field of sunspots from the toroidal field inside the sun.

A general discussion took place at the end of the session. A number of problems were formulated and these could serve as subjects for discussion as well as subjects for future studies.

The main of these problems were the following: 1) Is it necessary to consider all the gas in the galaxy as a continuous medium, or as separate clouds separated by a medium which has no effect upon the motion of the clouds? 2) Which is the principal mechanism of the dissipation of kinetic energy — the evolution of heat or the generation of cosmic rays — and what is the rate of dissipation? 3) What are the sources of kinetic energy? In particular, the differential rotation of the galaxy and the energy of hot stars were considered. 4) Does the dissipation decrease in a magnetic field? 5) The nature of the gaseous corona of the galaxy. 6) The character and the possible causes of formation of a regular field in spiral arms. 7) Is there, on the average, an equality between the magnetic and the kinetic energy densities? 8) Why is the gas density in the arms much higher than outside them?

The following session began with a paper by M. J. Seaton (England) entitled, "Present State of the Problem of Effective Cross-sections for Inelastic Collisions at Thermal Velocities.* Data were considered on the excitation of a sublevel of the ground state of hydrogen during interatomic collisions, excitation of the rotational structure of the H_2 molecule, the transitions 2s → 2p in a hydrogen atom on collision with a proton and an electron, the decrease of the two-quantum emission in the continuous spectrum of nebulae, charge exchange in O and H atoms, etc.

A number of inelastic cross sections for collisions of electrons with atoms were calculated using a modified Born approximation and the method of quantum defect. An approximate formula was obtained for the photoionization cross section of atoms.

Some of the following papers were concerned with ionization fronts. S. R. Pottasch (Holland) reported observations of bright rims in diffuse nebulae which are found in the neighborhood of dark matter. The position, orientation and density of these rims were compared with their form and the temperature of the star exciting the nebula (the hotter the star, the thicker the rims and the more distant are they from the star). The study was made of changes in the form, dimensions and the density of a rim as functions of the distance from the star and the size and density of the HII region. This author has established that the rims cannot be formed, as is often assumed, as a result of hydrodynamic instability of the Rayleigh-Taylor type. Instead, he calculated the effect of ionizing radiation on hydrogen. This leads to a density distribution in a rim (as a function of distance from the star) which is in agreement with observations. The density of dark matter behind a rim, calculated from this model, turns out to be very considerable.

F. D. Kahn (England) discussed a plane ionization front propagated in the neutral gas so that ionized gas flows away from the front with the velocity of sound. In this case pressure waves may appear in the neutral gas facing the front, and these have an effect on the motion of neutral gas, and may lead to instability if the amount of radiation reaching the front increases with time. However, absorption of the radiation in the ionized gas reduces this effect and instability can apparently only appear in special cases.

Motion in a cloud of ionized hydrogen,into which a star which ionizes the gas has penetrated,was considered by F. A. Goldsworthy for special cases of density distribution. The solution for a spherical wave in a cloud in which the density falls off as $1/r^2$ was based on a similarity between the flow pattern at different instants of time. Recombination was not taken into account. Various combinations of ionization and shock waves were discussed, e.g., a shock wave in a neutral gas and in an ionized gas. According to these calculations the velocity of the front should be of the order of 50 km/sec for an O5 star and about 25 km/sec for B2 star. However, the solution is not accurate because one cannot neglect recombination,since otherwise the heating of the central part ceases and the temperature falls to zero. A rough estimate shows that the velocity of propagation is almost unaffected when recombinations are taken into account. The other case which was considered was the propagation of a cylindrical front in the medium whose density falls off with distance from the axis as $1/r$. Recombinations and subsequent ionizations were taken into account. An estimate shows that the temperature of the ionized gas is not affected by taking motion into account. In this case it is possible to calculate the distribution of gas density in neutral and ionized regions. The velocity of the ionization front is proportional to the square root of the gas temperature and lies between 23 and 30 km/sec. Next, the structure of the ionization front was calculated,taking recombinations into account. The transition from neutral to ionized gas takes place at a distance of about 10^{-3} of the front radius.

The session of June 28 began with a discussion of the observable motion in nebulae. The paper by O. C. Wilson (U.S.A.) was concerned with internal motion in planetary nebulae. From the displacement of various forbidden lines one may obtain information on the motion of the various layers along the line of sight. It was concluded from such an analysis that a shell splits into two parts: the inner part accelerates towards the star and the outer part accelerates in the outward direction. The reason for this the author sees in the pressure of L_α radiation.* The turbulent motion is apparently small even in those objects in which noticeable fluctuations of brightness are present,so that these fluctuations are not the usual turbulent condensations. The form of the ellipses of distant nebulae is explained by a corresponding velocity distribution at the instant of ejection. This hypothesis was supported by observed spectroscopic properties of nebulae.

G. Munch (U.S.A.) investigated internal motions in the Orion and Crab nebulae. A special plate was prepared with 31 parallel slits placed at a distance of 1 mm from each other or 1".3 in the Coude focal plane. The plate was placed at the spectrograph slit and a single photograph gave ray velocities on an area 40" × 40" with a dispersion which was such that 1 km/sec could be measured. A comparison of the line widths of hydrogen and oxygen separated out the thermal component of the velocity, which for hydrogen was 8.9 km/sec, i.e., $T = 9700^\circ$, from the turbulent component, which was 7.4 km/sec. Statistical velocity fluctuations at different points on the image of the nebulae were compared with the theoretical distribution which one would expect if locally isotropic turbulence were present in the nebulae. The agreement turns out to be satisfactory. The absorption in the nebula is considerable and only the thin front layer can be observed directly. The reddening of stars in Trapezium shows that they are located close to the center of the nebula. Ray velocities of separate filaments were measured in the Crab nebula and this made it possible to determine the approximate position of the filaments in space, assuming that they all expand with the same velocity.

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* The role of L_α radiation in the splitting of a shell and the formation of double-shell nebulae was considered earlier by G. A. Gurzadian.

R. Minkowski (U.S.A.) reported observations of the spectra of the Cygnus Loop and some related nebulosities (NGC 6960 and NGC 6992). The inner part of the shell formed by the filaments expands with a velocity of about 65 km/sec, while the outer part expands with the velocity of about 115 km/sec. An analogous situation is found in the IC 443 nebula. The diameter of its main part is 40 parsec, while the velocity of expansion is 120 km/sec. The electron concentration was estimated by Osterbrock for bright filaments from the ratio of the components and was found to be of the order of 10^3 cm^{-3}. The mass of the separate filaments is of the order of $10^{-2} M_\odot$ and the total mass of the nebula reaches many solar masses. The conclusion as to the large mass of the shell of the supernova is of very great importance but it is necessary to verify once again the distance estimate.

The ratio H_α/H_β lies between 2.7 and 5.4 and its average value is 3.4. This quantity indicates that collisional excitation is by far the most probable type of excitation, while the lower limit of the ratio, which is reached in very small regions, corresponds to an electron temperature of about 100,000°. The high temperature of separate regions is also confirmed by the ratio of the lines 4363 and 5007 (OIII). The author considers that the Cygnus nebula is 46,000 years old and was formed as a result of an explosion of a supernova of type I.

In the discussion, M. J. Seaton stated that if in the calculation of recombinational emission of hydrogen the presence of different sublevels is taken into account then the ratio H_α/H_β will be considerably greater than that given in the old calculation of Menzel. This must be borne in mind in separating out the 'collisional' part of the emission.

H. Zanstra (Holland) considered the effect of gas and radiation pressure on the dynamics of planetary nebulae. For nebulae whose optical depth in the Lyman continuum in all directions is of the order of unity, gas pressure is more important than radiation pressure, and may give an observable expansion effect. If in some directions the optical depth is considerably greater than unity, the gas pressure tends to push the gas towards the star, forming currents which move in the inward direction and then become curved and emerge in more transparent places. The velocity of this flow may reach the observed value of 30 km/sec.

W. H. Bostick (U.S.A.) reported magnetohydrodynamic experiments with plasma in which the latter is ejected with a velocity of 300 km/sec in a strong discharge in a magnetic field. At the same time, a magnetic field is also induced in the plasma itself. The ejected plasmoids were deformed into loop-like forms. Two 'guns' were set up with parallel and anti-parallel fields. The plasmoids interacted, forming loops, spiral structures, etc. The author of this paper attempted to connect these structures with the galactic structure but did not give any calculations or detailed comparisons.

At the end of the conference, topics were suggested for the next symposium, which will take place in 1961 and will be devoted to the hydrodynamics of stellar atmospheres. The following problems will be considered:

1. Slow motion — photosphere, convective layer.
2. Fast motion — chromosphere, corona.
3. Separate objects — spots, protuberances, flares.
4. Pulsations of stars.
5. Aerodynamics of the upper layers of the earth's atmosphere.
6. Cosmogony — formation of planets, stars, galaxies.

7. Interplanetary medium.

8. Aerodynamic phenomena in stars — explosions of novae, etc.

9. Motion within the earth's nucleus.

In conclusion, the work of the conference was summarized. If the main theme of the first symposium was isotropic turbulence and shock waves, while the theme of the second symposium was the expansion of gas surrounding hot stars, the third symposium was characterized by discussions of the magnetic field, magnetohydrodynamics and dissipation calculations.

During the conference our delegation received much attention from the administrative staff of the Smithsonian Astrophysical Observatory and its Director, Prof. F. L. Whipple. We were shown the Massachusetts Institute of Technology and the Smithsonian Astrophysical Observatory, which is concerned with the organization of optical observations of the aritificial earth's satellites in the U.S.A. We were impressed during the visit to the radio astronomy station of Harvard University with its new 18-meter radio telescope and other equipment. The small staff, directed by T. K. Menon, is working very successfully in the 21 cm region, investigating separate objects, both galactic and extragalactic.

After the end of the symposium our delegation, together with Dr.Minkowski and his wife, flew from Boston to Los Angeles in order to visit major American observatories in California. At the airport we were met by Dr. I. Bowen, the director of the Mount Wilson and Mount Palomar Observatory and by Dr.J. Greenstein. We went by car to Pasadena, which is a suburb of Los Angeles. Los Angeles and its suburbs are contemporary American towns which grew up during the automobile years (and cannot even be thought of without them) and continue for almost a hundred kilometers, and do not have any of the usual means of transport except for very infrequent buses. Houses are mainly bungalows, small and designed for single families and separated by small gardens. Pavements are almost nonexistent.

The California Institute of Technology, which includes the Palomar Observatory, occupies a number of blocks. We were housed in the guest house, the Athenaeum. The next few days were occupied with visits to the Institute, conversations with the scientists, visits to Mount Palomar (400 km from Pasadena) and Mount Wilson (60 km), where we were able to see in detail the great telescopes and other equipment. We are not concerned in the present paper with a description of the observatories but we shall mention a few details. All the spectrographs are diffraction spectrographs with mirror optics including Schmidt plates. Frequently, the plate is in optical contact with the surface of the grating. Because of the curvature of the field, plates are made from thin glass which can be easily bent in a cassette. The plates are kept in cooled containers, ordinary at T = 1-2°C, infrared at 10°C. Before use, the plates are sensitized by ammonia in the case of panchromatic plates, and by thermal processing in the case of nonsensitized plates; they are kept two or three days at 50°C or one day at 65°C. Much attention is paid to development: all the photographic rooms are air conditioned and kept at a constant temperature; all the dishes are of stainless steel. To develop large 48" Schmidt plates a special rotating bath was made which has a vibrating axis in which, after many experiments, standing waves were eliminated. The developer is used only once; it is kept in large bottles at low temperature and after use it is disposed of.

Diffraction gratings, spectrographs, fine and medium optics, many instruments and subsidiary equipment are prepared in workshops which have a few highly qualified mechanics and good equipment. Much attention is paid to the well-being and recreation of the workers who work at Pasadena for a few days at a time. Large observatories have adopted the practice of inviting astronomers from other towns and countries, who then live in Pasadena for a few months, carry out some studies and then publish on behalf of the Observatory. This practice increases the possibilities of the observatories at a relatively small staff, brings in new ideas and ensures the full use of the equipment.

The staff of the Observatory, beginning with the Director, Dr. Bowen, made every effort to make our visit a pleasant one. Dr. Bowen and Dr. Babcock often drove us to observatories, gave very detailed explanations and answers to questions. R. Minkowski, J. Greenstein, W. Baade, W. Baum, A. Deutsch, A. Code, D. Osterbrock, and others on the staff of the Observatories showed us many interesting data and described results of their work. Miss H. Swope took much trouble in connection with our visits.

On July 8 Dr. A. Deutsch drove us to the Lick Observatory (750 km). The observatory is situated in a distant mountainous place at an altitude of 2000 meters, and all its staff lives there. We were met very cordially by the staff of the observatory and above all, its Director, Dr.C. Shane, who was our host. During the tour of the observatory C. Shane, N. Mayall, G. Herbig, S. Vasilevskis and others showed us details of the instruments, particularly the remarkable 3m reflector, talked about their work, and showed photographs, spectrograms and other material.

After a short but very interesting stay at the Lick Observatory, Dr. G. Herbig drove us back to Los Angeles, from where on July 11 we flew to Stockholm and then home.

S. B. Pikel'ner

Received August 6, 1957

ON THE INTERNATIONAL COOPERATION FOR THE STUDY OF FLARE VARIABLE STARS

M. Petit

The idea of an international cooperation for the study of the phenomena of outbursts in stars was first proposed by the author in a note published in 1949 [1]. This proposal was afterwards repeated and developed independently by several persons.

Up to the present time, this cooperation has been effectively realized only between several observers who, while interested in the subject and in an exchange of information, did not consider the general program for the investigation necessary. This cooperation can be extended to advantage; it appears that through the use of the resources available to each observer carrying out a definite program of observation and through a rapid dissemination of the information collected* it should be possible to obtain conclusive results for a number of difficult problems.

The investigation of unstable stars as a class is interesting for three reasons:

a) the investigation of luminosity variations;

b) the spectroscopy and study of stellar atmospheres;

c) the investigation of the structure of the galaxy and stellar evolution.

Photometric Observations

Because of the exceptional rapidity of luminosity changes, special problems are encountered** which are difficult to solve and which sometimes make classical methods of observation unusable.

1. Visual observations. The usual method (called the comparison method) is not applicable: it is necessary to carry out continuous visual observations which are fairly fatiguing if the observations are of long duration. If the observations are accompanied by estimation of luminosity, in this case made according to the method of Argelander, then it is necessary to guard against eye fatigue, as well as Purkinje's phenomenon.

Accurate measurements are not possible with visual observations, but their advantage lies in that they are accessible to a large number of amateur, as well as professional, astronomers, who do not possess special instruments. In the case of the observation of outbursts, they lead to a better estimate of amplitude and duration than most photographic methods.

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*It should be pointed out that Professor B. V. Kukarkin in 1956 proposed a general program for the investigation of bright stars and variables of all types and, in particular, of the long-period variables.

**It is to be noted that the pre-maximum rate of increase of luminosity in the case of N Aql 1918 was 0.03 mag./min or 0.0005 mag./sec. However, during the outburst of UV Ceti on September 25, 1952, the luminosity increased by 0.25 mag./sec, i.e., 500 times as fast as in the case of N Aql.

2. Photographic observations. Three methods can be used:

a) Numerous short exposures: some authors (Leiten, Lippincott, de Kock) used a collection of plates, obtained for the determination of parallax and proper motion, for the study of outbursts, but this method is open to error: the exposures are usually too long to obtain the behavior of the "luminosity-increase phase" (which in the case of the variables of the UV Ceti type lasts for only several seconds), but, on the other hand, they are too short for a reliable statistical investigation of outbursts – 259 plates of EQ Pegasi investigated at the Sproul observatory by Miss Lippincott [2] cover only 3^h30^m of the exposure (the average exposure is 45^h), i.e., possibly less than a tenth of the time interval between successive outbursts.

However, the use of this method might be of interest in the case of photographs of regions containing nebulae: the work of Haro, Rosino, and Wenzel has shown that it is then possible to find stars with rapid luminosity changes in a series of short-exposure plates obtained for a comparatively narrow region.

b) Long exposures (10–60 min) with wide-field plates. This method has the advantage that it is possible to employ "Sky Patrol" plates which are used at the various observatories for the investigation of variables or comets. Wenzel* reports that the results obtained at Zonnerberge were found to be disappointing; the large number of plates obtained with the "Ernostar" objective having a wide field (30° × 30°) led to the discovery of only 200 variables, but of these there was only one possible flare star (KZP 5235). On the other hand, Weber and I [3] have investigated approximately 150 plates for 14 fields, obtained with the help of two small Russell-Taylor and Tessar objectives, and in spite of the small number of plates investigated we have found two outbursts each of UV Ceti and BD + 55° 1823 and have suspected the variability of BD + 13° 2618.

This method can be fruitful for the investigation of newly found variables and for the study of outburst frequency but, on the other hand, it is unsuitable for the observation of outburst development.

c) Continuous registration: Weber** has proposed the use of cine-film,which makes it possible to discover changes in luminosity and from the known velocity of the film (5 mm/sec) to obtain the history of an outburst. This method can be used,but it is not very convenient since it requires the use of fairly powerful optical systems.

3. Photoelectric observations. Photoelectric registration is undoubtedly the best solution, both because of the accuracy attainable, as well as because of the possibility of making measurements in several colors. As has been shown by Rock, it is possible to obtain very good light-curves for the outbursts.

Astrophysical Observations

With the use of low-dispersion instruments to obtain a series of spectra it would be possible to discover outbursts and to study the changes in the spectra which occur without outbursts [4].

With the help of large instruments, it would also be desirable to obtain detailed spectra suitable for microphotometric study; the quantitative data obtained would make it possible to investigate the observed phenomena theoretically.

Photoelectric measurements of the color index and the variations of the color index could also provide interesting information which would be obtained in a simpler way than the information from the high-dispersion spectra.

Finally, micrometric measurements of the proper motion and parallax are useful for two reasons: to obtain information on the absolute magnitudes of the variables and on the structure of the galaxy in the vicinity of the sun.

Problems Which Have to be Solved

We do not as yet know anything about unstable stars as a class; the data obtained until now by permanent observers are in most cases only qualitative. A systematic and coordinated research program is necessary.

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*Private communication.

**Unpublished.

From a systematic investigation of unstable stars it almost certainly follows that all of the near-red dwarfs that exhibit hydrogen emission are variable stars. A systematic photometric and spectroscopic investigation of these stars is desirable; however, they are few in number (about 75). For this, it is necessary:

a) to obtain a sufficiently large number of photographs of these stars with the help of the photographic methods listed above;

b) to carry out investigations with the help of old photographic plates;

c) to obtain a large number of spectra with the help of low-dispersion instruments.

It also appears to be profitable to study and consider separately cases of "transient" stars which appear in one or two plates obtained simultaneously but are absent in others. Several cases of this type have already been investigated by Hoffleit [5] and myself [6].

Dwarfs of the T Tauri type are of particular interest. At the present time they are the subject of comprehensive investigations by several workers [7]. It would also be profitable to begin the fairly difficult investigation of unstable stars in the young clusters of the type of the Orion cluster or of NGC 2264, for example, NGC 2244, NGC 6530 (M 8), NGC 6611 (M 16), and IG 5146.

Walker [8] has made a photoelectric investigation of several post-novae and has established that in some of them, for example, T CrB, GK Per, and V 603 Aql, very rapid changes and a considerable amplitude are to be found. It is desirable to extend these observations to all the sufficiently bright post-novae, as well as to certain types of nova-like stars and to variables of the U Gem type,which have been studied by Grant [9] and the author.

Some problems, discussed earlier* and associated with the observation of the frequency, the amplitude and duration of outbursts, the presence of secondary variations, and the changes in the spectra require systematic observations of the most typical unstable stars and an extension of photoelectric measurements. Only when sufficiently large extensive observational data become available will we be able to confirm the hypotheses made.

It would be of interest to increase the number of measurements of the proper motions and parallaxes of the near-red dwarfs and the T Tauri stars,since a knowledge of their absolute magnitudes is of great importance to cosmogony. In addition, repeated measurements of the numerous binaries known among these stars are necessary since the only unstable star whose mass we know with some accuracy at the present time is DO Cep (Kru 60 B).

Finally, the measurement of radial velocities associated with micrometric measurements will give us information on the kinematical properties of faint stars.

Ambartsumian [10] has recently shown the importance of a systematic study of UV Ceti and T Tauri stars to the investigation of stellar evolution, but it should be noted that while the T Tauri stars are being studied to some extent at the present time, the UV Ceti stars have almost been completely neglected.

Proposal for International Cooperation

After the general problems have been outlined, it is possible to determine the basis for international cooperation for the study of these variable stars.

1) It is possible to call on a sufficiently large number of observers possessing a variety of instruments because all sufficiently powerful instruments can be used for this work. Differences between the facilities available to the different observers, however, should not be an obstacle to the use of the data obtained.

2) It would be desirable**, if possible, for the observers to be well distributed geographically so that, if necessary, observations of one variable star could be made at fairly short intervals 24 hours a day.

3) Each observer will, to some extent, act independently, but it is necessary that the data be sent to a coordinating center and that the main directions be determined and given to all the participants in this work.

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*In our two papers in Peremennye Zvezdy [Variable Stars].

**However, this is very difficult to realize in practice.

4) The collection of data at the coordinating center will make it possible to publish and disseminate rapidly the results obtained.

5) In order to establish the connection between outbursts on stars and solar chromospheric phenomena, it is necessary that relations be established with observatories carrying out solar observations.

6) The theoretical investigation of the phenomenon of outbursts is at the present time premature because we do not have adequate quantitative data. It will only be possible when a sufficient amount of observational data has been collected.

I am grateful to Professor B. V. Kukarkin, president of the 27th International Astronomical Union Committee, who asked me to write this note, which is an expanded version of a circular sent in September, 1956 to several observatories.

Paris Observatory
Longitude Bureau

Received June 17, 1957

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SOVIET ASTRONOMY – AJ

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SIGNIFICANCE OF ABBREVIATIONS MOST FREQUENTLY ENCOUNTERED IN SOVIET ASTRONOMY PERIODICALS

AN SSSR	*Academy of Sciences, USSR*
FIAN	*Physics Institute, Academy of Sciences USSR*
GAFI	*State Astrophysical Institute*
GAISh	*P. K. Shternberg State Institute of Astronomy*
GAO	*Main Astronomical Observatory*
Gidrometizdat	*Hydrometeorological Press*
GITI	*State Scientific and Technical Press*
GITTL	*State Press for Technical and Theoretical Literature*
GOI	*State Optical Institute*
GONTI	*State United Scientific and Technical Press*
Gosfizkhimizdat	*State Physical Chemistry Press*
GOST	*All-Union State Standard*
Goztekhizdat	*State Technical Press*
GTTI	*State Technical and Theoretical Press*
IF KhI	*Institute of Physical Chemistry Research*
IFP	*Institute of Physical Problems*
IL	*Foreign Literature Press*
IPF	*Institute of Applied Physics*
IPM	*Institute of Applied Mathematics*
ISN (Izd. Sov. Nauk)	*Soviet Science Press*
IIaP	*Institute of Nuclear Studies*
Izd	*Press (publishing house)*
KISO	*Solar Research Commission*
LFTI	*Leningrad Institute of Physics and Technology*
LIM	*Leningrad Institute of Metals*
LITMiO	*Leningrad Institute of Precision Instruments and Optics*
Mashgiz	*State Scientific-Technical Press for Machine Construction Literature*
MATI	*Moscow Aviation Technology Institute*
MGU	*Moscow State University*
Metallurgizdat	*Metallurgy Press*
MOPI	*Moscow Regional Pedagogical Institute*
NIAFIZ	*Scientific Research Association for Physics*
NIFI	*Scientific Research Institute of Physics*
NIIMM	*Scientific Research Institute of Mathematics and Mechanics*
NII ZVUKSZAPIOI	*Scientific Research Institute of Sound Recording*
NIKFI	*Scientific Institute of Motion Picture Photography*
OIIaI	*Joint Institute of Nuclear Studies*
ONTI	*United Scientific and Technical Press*
OTI	*Division of Technical Information*
OTN	*Division of Technical Science*
RIAN	*Radium Institute, Academy of Sciences of the USSR*
Ts NIIGAiK	*Central Scientific Research Institute of Geodetics, Aerial Survey, and Cartography*
URALFTI	*Ural Institute of Physics and Technology*
VAGO	*All-Union Astronomic and Geodetic Society*
VNIIM	*All-Union Scientific Research Institute of Meteorology*

NOTE: Abbreviations not on this list and not explained in the translation have been transliterated, no further information about their significance being available to us.—*Publisher.*

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